TRISEP 2021 : $\underbrace{\text{Standard Model}}_{\text{SM}}$

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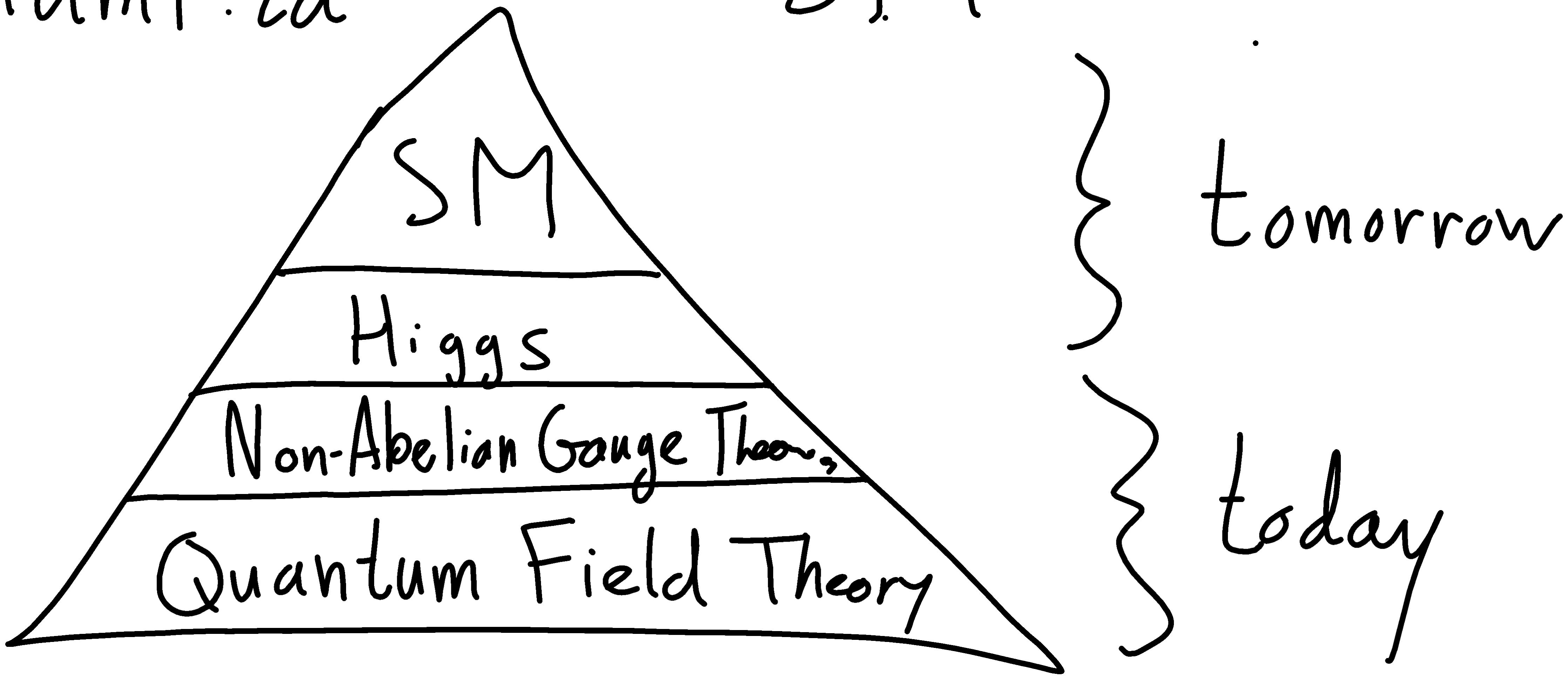


Natural Units: $\hbar = 1 = c$

$\hbar \sim E \cdot T \qquad c \sim L/T$

$\boxed{\begin{array}{l} \hbar c = 197 \text{ MeV fm} \\ \text{fm} = 10^{-13} \text{ cm} \end{array}}$

energy ~ mass ~ momentum

→ eV = "electron volt"

keV = 10^3 eV

MeV = 10^6 eV

GeV = 10^9 eV

e.g. particle

$E = m$

Special Rel.

$\hookrightarrow$ invariance under $\underbrace{\text{rotations + boosts}}_{\text{"Lorentz transformations"}}$

Lorentz scalar: doesn't transform under Lorentz

Lorentz vector := "4-vector"

4×4 Lorentz matrix

$$a^\mu \longrightarrow a'^\mu = \Lambda^\mu{}_\nu \, a^\nu = \sum_{\nu=0}^{3} \Lambda^\mu{}_\nu \, a^\nu$$

summed over, $\nu = 0,1,2,3$

position: $x^\mu = (t, x, y, z) = (t, \vec{x})$

momentum: $p^\mu = (E, p^x, p^y, p^z) = (E, \vec{p})$

$$E = \sqrt{m^2 + \vec{p}^{\,2}}$$

$$a \cdot b = \eta_{\mu\nu}\, a^\mu b^\nu \, , \quad \eta_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

sum

$$= a^0 b^0 - \vec{a} \cdot \vec{b}$$

Lorentz scalar $= a \cdot b = a' \cdot b'$

Define $a_\mu = \eta_{\mu\nu} a^\nu = \begin{cases} a^0 & ; \mu = 0 \\ -a^i & ; \mu = i \end{cases}$

$$\partial_\mu = (\partial_t, \partial_x, \partial_y, \partial_z) = (\partial_t, \vec{\nabla})$$

$$\partial_t = \frac{\partial}{\partial t}, \quad \partial_x = \frac{\partial}{\partial x}, \quad \partial_y = \frac{\partial}{\partial y}, \quad \partial_z = \frac{\partial}{\partial z}$$

$$\partial^2 = \eta^{\mu\nu} \partial_\mu \partial_\nu = \partial_t^2 - \vec{\nabla}^2$$

Tensor: $T^{\mu\nu} \to T'^{\mu\nu} = \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta T^{\alpha\beta}$

Quantum Field Theory (QFT).

Variables are fields on spacetime

$\phi_i(t, \vec{x}) = \phi_i(x) =$ value at every spacetime point

Define theory with "action".

$$\underbrace{S}_{\text{action}} = \underbrace{\int d^4x}_{= \int dt \int dx \int dy \int dz \text{, over all spacetime}} \, \mathcal{L}(\phi_i)$$

$\mathcal{L}$ = Lagrangian, function of fields.

Scalar: $\phi(x) \to \phi'(x') = \phi(x)$, under $x^\mu \to \Lambda^\mu{}_\nu x^\nu = x'^\mu$

Vector: $A^\mu(x) \to A'^\mu(x') = \Lambda^\mu{}_\nu A^\nu(x)$

Fermion: $\psi_a(x) \to \psi'_a(x') = M_a{}^b(\Lambda)\, \psi_b(x)$

a = 1, 2, 3, 4 = "spinor indices"

sum on these

$$M(\Lambda) = \text{function of } \Lambda.$$

$$M(\mathbb{1}) = \mathbb{1}, \quad M(\Lambda_1)M(\Lambda_2) = M(\Lambda_1\Lambda_2)$$

Use these to write Lagrangians.

$$S = \int d^4x \, \mathcal{L}(\phi)$$

same point

$$\mathcal{L} = -\frac{1}{2} m^2 \phi(x) \cdot \phi(x) = \text{local}$$

$$\mathcal{L} = -\frac{1}{2} m^2 \phi(x) \phi(y) = \text{non-local}$$

different

Need: 1. "unitary" $\rightarrow$ $\mathcal{L}$ is real

2. "local" $\rightarrow$ $\mathcal{L}$ depends on ϕ at same x points.

3. $\mathcal{L}$ should Lorentz invariant

Recipe for QFT

1. Start with quadratic terms, identify mass + kinetic
2. Make sure kinetic are "canonical", masses diagonal.
3. Now add higher order terms, and identify them as interactions.

e.g. Free scalar field

$\phi(x) =$ scalar field = real valued

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$$

$$(\partial\phi)^2 = \eta^{\mu\nu}\partial_\mu\phi \cdot \partial_\nu\phi = (\partial_t\phi)^2 - \vec{\nabla}\phi \cdot \vec{\nabla}\phi$$

$(\partial\phi)^2$: kinetic term

$\frac{1}{2}m^2\phi^2$: mass term

$\mathcal{L}$ describes a spin $s=0$ particle with mass

$$m_{phys} = \sqrt{m^2} = m.$$

e.g. Complex scalar field

$$\mathcal{L} = |\partial \Phi|^2 - m^2 |\Phi|^2 \quad \to \text{real, local, L.I.}$$

$$|\partial\Phi|^2 = \eta^{\mu\nu} \partial_\mu \Phi^\dagger \partial_\nu \Phi, \qquad |\Phi|^2 = \Phi^\dagger \Phi$$

$\to$ describes: $s=0$ particle of mass m

$s=0$ anti-particle of mass m

$$\Phi(x) = \frac{1}{\sqrt{2}}\left[\phi_1(x) + i\,\phi_2(x)\right], \quad \phi_1, \phi_2 = \text{real scalars}$$

e.g. Vector Field

$$\mathcal{L} = -\tfrac{1}{4} \underbrace{F_{\mu\nu} F^{\mu\nu}}_{\text{kinetic}} + \underbrace{\tfrac{1}{2} m^2 A_\mu A^\mu}_{\text{mass}}, \text{ plus constraint } \partial_\mu A^\mu = 0.$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu =$ "field strength"

→ describes $s=1$ particle with mass m

↳ 3 degrees of freedom

But A_μ seems to have 4!?

↳ removed with constraint

e.g. Dirac fermion

$$\mathcal{L} = \underbrace{\bar{\Psi} i\gamma^\mu \partial_\mu \Psi}_{\text{kinetic term}} - \underbrace{m\bar{\Psi}\Psi}_{\text{mass term}}$$

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}, \quad \bar{\Psi} = \psi^\dagger \gamma^0 = (\bar{\psi}_1, \bar{\psi}_2, \bar{\psi}_3, \bar{\psi}_4)$$

$\gamma^\mu = 4 \times 4$ gamma matrices, $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$.

→ desribes $s = \frac{1}{2}$ fermion, $s = \frac{1}{2}$ anti-fermion, of mass m.

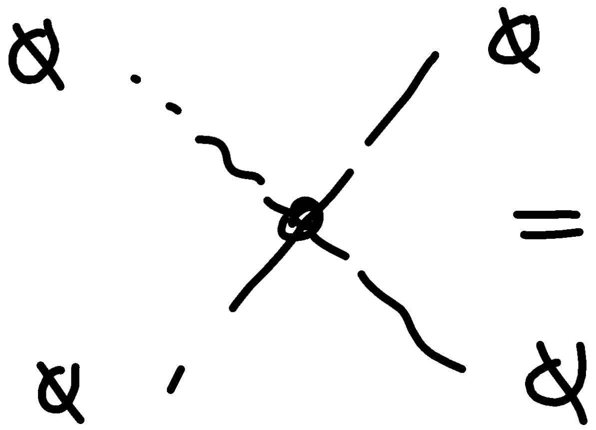
Now add higher orders for interactions.

e.g. $$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2}_{\text{"free particle"}} - \underbrace{\frac{\lambda}{4!}\phi^4}_{\text{interaction}}$$

Treat ϕ^4 as a perturbation on the free theory.

We do this with "Feynman Rules".

With ϕ^4 term, we now have $\phi+\phi\to\phi+\phi$ scattering.

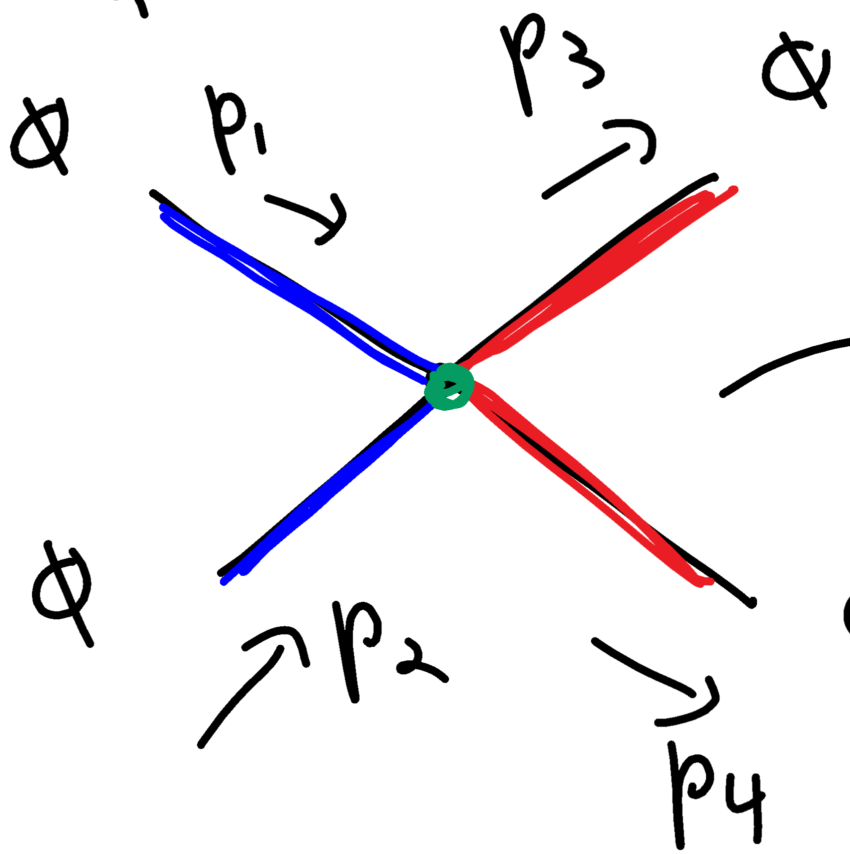
$\frac{\lambda}{4!}\phi^4 \to$ 

$=$ vertex with 4 phi "ϕ" particles.

Feyman Diagram:

$\phi(p_1)+\phi(p_2)$

$\to \quad \phi(p_3)+\phi(p_4)$



$\longrightarrow$ time

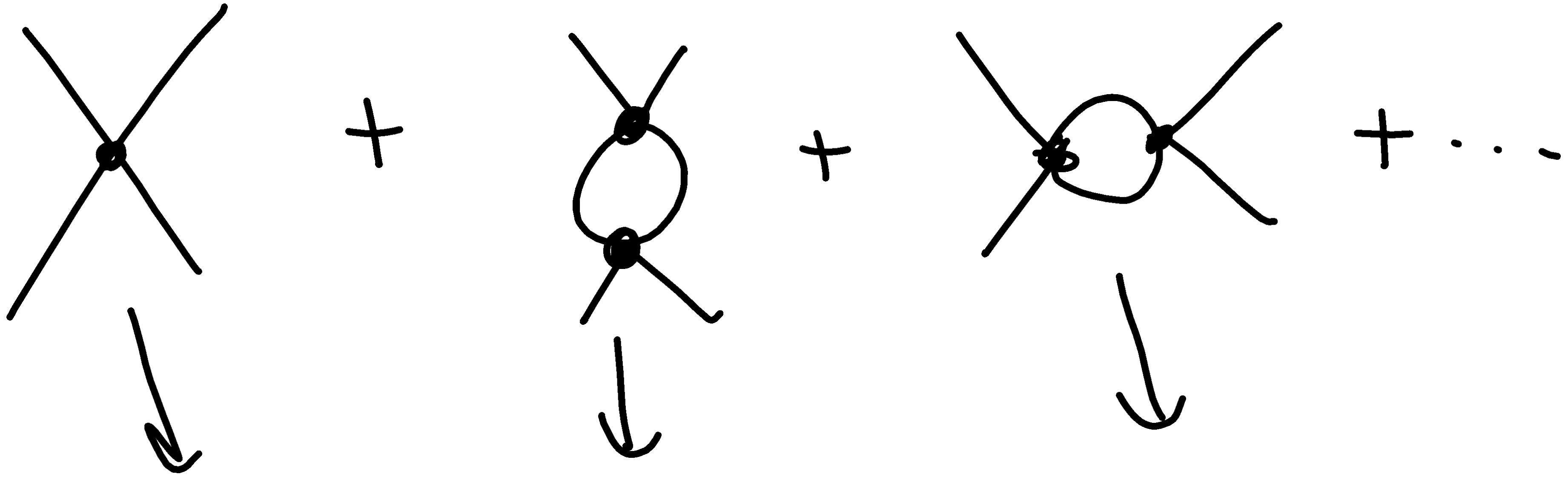
— = incoming external leg

— = outgoing ext. leg

$\times$ = interaction vertex

$$M_{fi} = \langle f \,|\, i \rangle = \lambda$$

& + & → & + &



$$\mathcal{M}_{fi} \sim \lambda \qquad \lambda^2 \qquad \lambda^2 + \dots$$

As long as λ is "small", lowest orders are the most important!

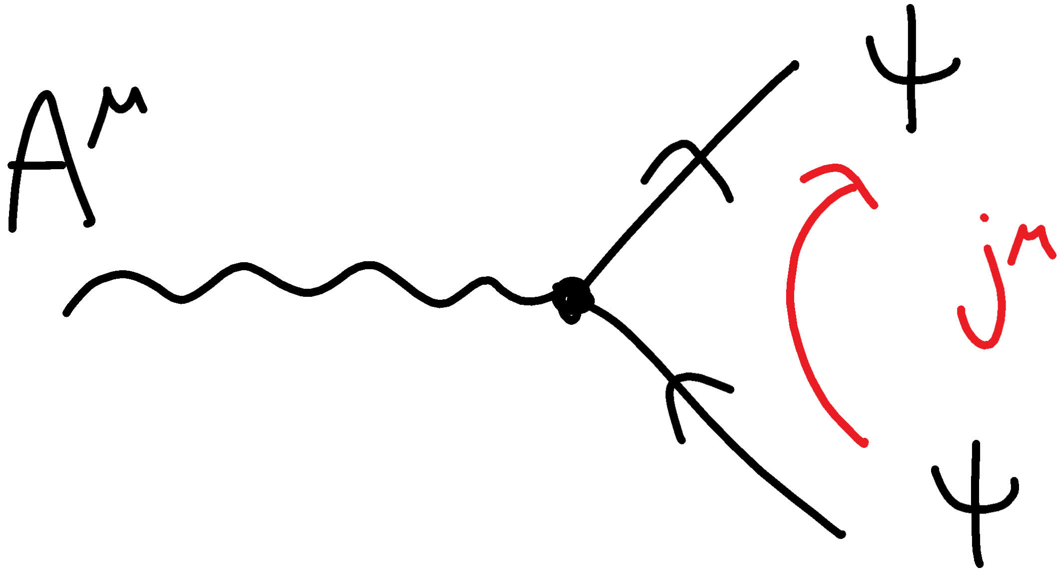
e.g. QED = quantum electrodynamics

$\psi \sim$ electron

$A^\mu \sim (\phi, \vec{A})$ = em scalar + vector potentials

$$\mathcal{L} \supset -e\overset{\text{em charge of fermion } \psi}{Q}\, \bar{\psi}\gamma^\mu\psi \cdot A_\mu$$

$\rightarrow$ vertex:



$$= -\underbrace{ieQ}\gamma^\mu$$

Symmetries

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

$\rightarrow$ symmetry under $\phi \rightarrow -\phi$

"

transformation on fields s.t. $\mathcal{L}$ is unchanged.

$\longrightarrow$ implies $\phi + \phi \rightarrow \phi + \phi$ is allowed

$\underset{(-1)^2}{\phi + \phi} \rightarrow \underset{(-1)^3}{\phi + \phi + \phi}$ is not allowed

$(-1)^2 \neq (-1)^3$

e.g. $\mathcal{L} = \frac{1}{2}\left[(\partial\phi_1)^2 + (\partial\phi_2)^2\right] - \frac{1}{2}m^2(\phi_1^2 + \phi_2^2)$

$= \frac{1}{2}(\partial\phi^t)\cdot(\partial\phi) - \frac{1}{2}m^2\phi^t\phi$

$\phi = \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}$

orthogonal 2x2 matrix

Symmetric under $\phi \to \phi' = \underset{\text{orthogonal } 2\times 2 \text{ matrix}}{O}\phi$ $\qquad O^t O = \mathbb{1} = O O^t$

$m^2\phi^t\phi \to m^2(O\phi)^t(O\phi) = \phi^t \underbrace{O^t O}_{=\mathbb{1}}\phi \cdot m^2 = m^2\phi^t\phi$

$$\Theta = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \quad , \alpha \in [0, 2\pi)$$

$\Rightarrow$ "continuous symmetry"

$$\mathcal{L} = \bar{\Psi} i \gamma^\mu \partial_\mu \Psi - m \bar{\Psi} \Psi$$

$$\begin{cases} \Psi \to e^{i\alpha} \Psi \\ \bar{\Psi} \to \bar{\Psi} e^{-i\alpha} \end{cases} , \quad \alpha = \text{constant}$$

$$m \bar{\Psi} \Psi \to m \left(\bar{\Psi} \underbrace{e^{-i\alpha} \right) \left(e^{i\alpha}}_{=1} \Psi \right) = m \bar{\Psi} \Psi$$

QED

photon A^μ kinetic

interaction

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_i \left[\bar{\Psi}_i \, i\gamma^\mu (\partial_\mu + ieQ_i A_\mu)\Psi_i - m_i \bar{\Psi}_i \Psi_i \right]$$

kinetic Ψ

mass Ψ

Symmetry : $\begin{cases} \Psi_i \to e^{i\alpha Q_i} \Psi_i \\ A_\mu \to A_\mu \end{cases}$, but only for $\alpha =$ constant

What about $\alpha = \alpha(x)$?

$\hookrightarrow$ not a symmetry : $\bar{\Psi} i\gamma^\mu \partial_\mu \Psi \to \bar{\Psi}' i\gamma^\mu \partial_\mu (e^{i\alpha Q} \Psi)$

Fixed by gauge invariance!

BUT: do have an invariance under

$$\begin{cases} \psi_i \to e^{i\alpha Q_i} \psi_i & \to \mathcal{L} \text{ is invariant} \\ A_\mu \to A_\mu - \frac{1}{e} \partial_\mu \alpha & \end{cases}$$

Show that: $D_\mu \psi_i = (\partial_\mu + ieQ_i A_\mu)\, \psi_i$

$$\longrightarrow e^{i\alpha Q_i} (D_\mu \psi)$$

$F_{\mu\nu}$ is also invariant

"Gauge invariance"

↳ interpret as equivalence relation

→ any two configurations related by a gauge transformation describe the same physical state.

$$A^{\mu} = (\phi, \vec{A})$$

QED: $\psi \to e^{i\alpha Q}\psi$

↳ rephasing

"Group" of rephasings $=$ "$U(1)$" $= 1\times 1$ unitary transformt

$U^\dagger U = \mathbb{1} = U U^\dagger$, satisfied by $e^{i\alpha Q}$.

Generalize $U(1)$ to $SU(2)$.

$$U(\alpha^a) = e^{i\alpha^a t^a} \quad , \quad a = 1,2,3$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$t^a = \frac{\sigma^a}{2} \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\swarrow$

2x2 unitary matrices

"S" $\rightarrow$ $\det(U) = 1$.

$$e^{i\alpha^a t^a} = \sum_{n=0}^{\infty} \frac{(i\alpha^a t^a)^n}{n!} \qquad [t^a, t^b] = i\epsilon^{abc} t^c.$$

Can also generalize to other groups.

e.g. $SU(N) = N \times N$ unitary matrices wrt $\det = 1$.

$$\mathcal{U}(\alpha^a) = e^{i\alpha^a t^a}, \quad t^a = \text{Hermitian "generators"}$$

$N^2 - 1$ generators

$$[t^a, t^b] = i f^{abc} t^c$$

f^{abc} = structure constants.