Dark Matter Intro

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Dark Matter

- How do we know it exists?
- How is it distributed?
- How does it interact?
- What is it?
- How was dark matter produced?

How do we know it exists?

- Dark matter is inferred from its gravitational effects on galactic and cosmological scales.

Observations: $G_{\mu\nu} \neq 8\pi G_N T_{\mu\nu}$

$T_{\mu\nu}$ **Dark Matter**
Dark Energy

$G_{\mu\nu}$ **Modify gravity**

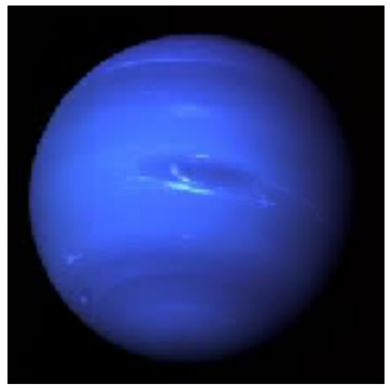
How do we know it exists?

- Dark matter is inferred from its gravitational effects on galactic and cosmological scales.

Observations: $G_{\mu\nu} \neq 8\pi G_N T_{\mu\nu}$

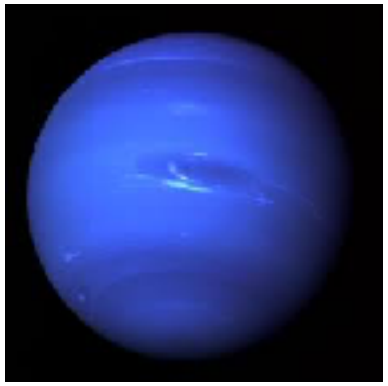
$T_{\mu\nu}$	**Dark Matter** **Dark Energy**	**Standard Cosmological Model**
$G_{\mu\nu}$	**Modify gravity**	

A parable: Neptune and Vulcan



- In the 1800's the perihelion of Uranus' orbit was not consistent with the known objects in the solar system.
- Dark Matter: Adams and Le Verrier postulated a new planet, Neptune, to explain the orbit of Uranus — it was found where predicted just months later!
- Modified Gravity: Building on this success, Le Verrier postulated a new planet, Vulcan, to explain the precession of Mercury's perihelion. Wrong! The correct explanation is GR.

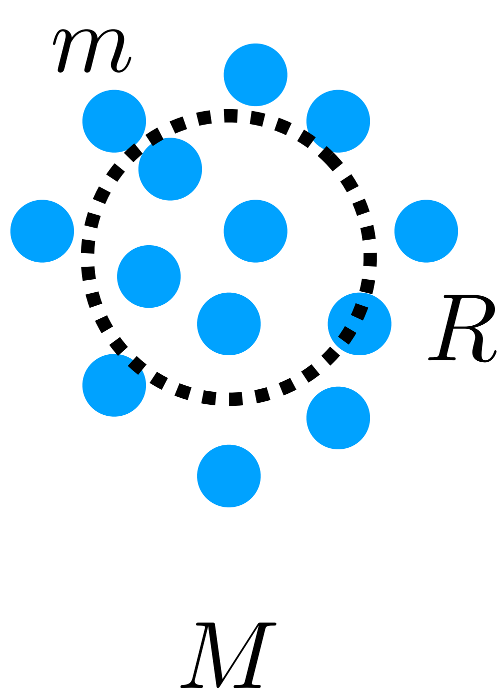
A parable: Neptune and Vulcan



Keep an open mind!

An example: velocity dispersion

- Given a collection of point masses in orbit around their centre of mass, we can apply the Virial Th'm to obtain the total mass:



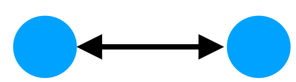
$$\langle KE \rangle = -\frac{1}{2}\langle PE \rangle$$

$$\frac{1}{2}m\langle v^2 \rangle \simeq \frac{1}{4}\frac{G_N M m}{R}$$

$$M = \frac{2R\langle v^2 \rangle}{G_N}$$

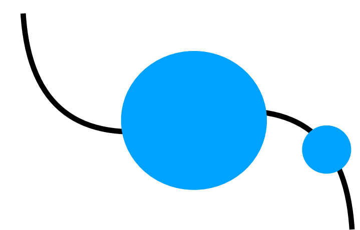
Aside: astronomical distances

nearest star



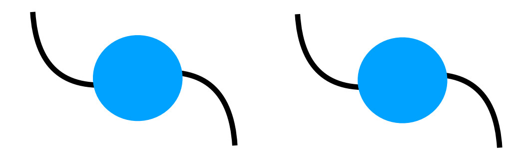
$1\ \mathrm{pc} = 3.25\ \mathrm{ly} = 3 \times 10^{16}\ \mathrm{m}$

galactic center



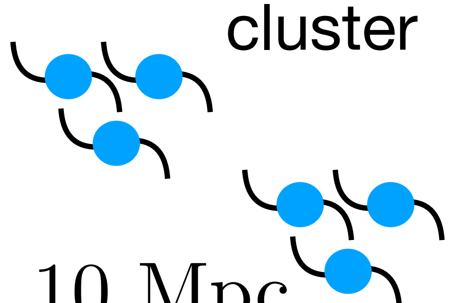
$10\ \mathrm{kpc}$

nearest galaxy



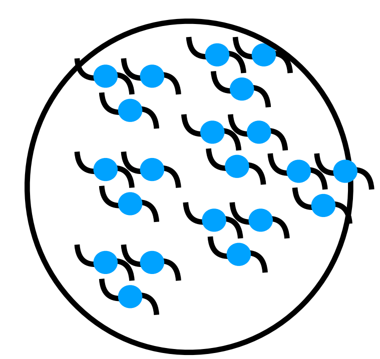
$1\ \mathrm{Mpc}$

nearest galaxy cluster



$10\ \mathrm{Mpc}$

observable Universe



$10\ \mathrm{Gpc}$

Universe.........

An example: velocity dispersion

$$M = \frac{2R\langle v^2 \rangle}{G_N}$$

$$G_N = 4.32 \times 10^{-3}\ (\mathrm{km/s})^2 \mathrm{pc}/\mathrm{M}_\odot$$

Globular Clusters

$$R \sim 10\ \mathrm{pc}$$

$$\langle v^2 \rangle \sim 10\ (\mathrm{km/s})^2$$

$$M \sim 10^4\ \mathrm{M}_\odot$$

Compare with Luminosity:

$$L \sim 10^4\ \mathrm{L}_\odot$$

Can account for mass from stars!

Galaxy Clusters

$$R \sim 2\ \mathrm{Mpc}$$

$$\langle v^2 \rangle \sim 10^6\ (\mathrm{km/s})^2$$

$$M \sim 10^{15}\ \mathrm{M}_\odot$$

Compare with Luminosity:

$$L \sim 10^{13}\ \mathrm{L}_\odot$$

Can't account for mass from stars!

An example: velocity dispersion

- A solution: There is ~100 times more non-luminous matter (dark matter) inhabiting galaxy clusters than luminous matter. If this is distributed uniformly inside the cluster, then the amount inside globular clusters is negligible.

- Another solution: Gravity behaves differently in clusters of galaxies than clusters of stars.

Evidence for dark matter

- There is evidence for dark matter across a wide variety of scales:

dwarf galaxies

$M \sim 10^9\ \mathrm{M}_\odot$

$R \sim \mathrm{kpc}$

galaxies

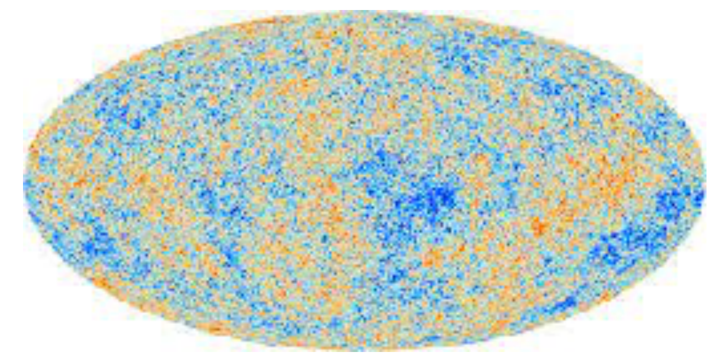
$M \sim 10^{12}\ \mathrm{M}_\odot$

$R \sim 10 - 100\ \mathrm{kpc}$

galaxy clusters

$M \sim 10^{15}\ \mathrm{M}_\odot$

$R \sim 1 - 10\ \mathrm{Mpc}$



observable Universe

$R \sim 100 - 10^4\ \mathrm{Mpc}$

← **astrophysics: non-linear**

→ **cosmology: linear**

Modelling the Universe

- Much of cosmology is described by General Relativity and Relativistic Hydrodynamics of perfect fluids.
- The fundamental variables:

$g_{\mu\nu}(x,t)$	$\rho_i(x,t)$	$u^\mu(x,t)$
metric	fluid densities	fluid velocities

- The laws: Einstein and continuity equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad \nabla_\mu T^{\mu\nu} = 0 \qquad T_{\mu\nu} = \sum_i (\rho_i + p_i)\, u_{i\,\mu} u_{i\,\nu} + p_i g_{\mu\nu}$$

Set of coupled PDE's -- need initial conditions!

The Linear Universe

- For much of the history of the Universe:

$$g_{\mu\nu}(x,t) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(x,t)$$

$$\rho_i(x,t) = \bar{\rho}_i(t) + \delta\rho_i(x,t) \qquad \delta = \text{ small}$$

$$u^\mu(x,t) = \bar{u}^\mu + \delta u^\mu(x,t)$$

- Why was the universe so nearly homogeneous?
- For today: this is an extraordinary convenience!

Universe

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

→ Homogeneous Background

$$\bar{G}_{\mu\nu} = 8\pi G \bar{T}_{\mu\nu}$$

→ Fluctuations

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$$

The Linear Universe

- The homogeneous Universe:

$$p_i = w_i \rho_i \qquad \bar{\rho}_i(t_0)$$

Types of fluids — Densities today

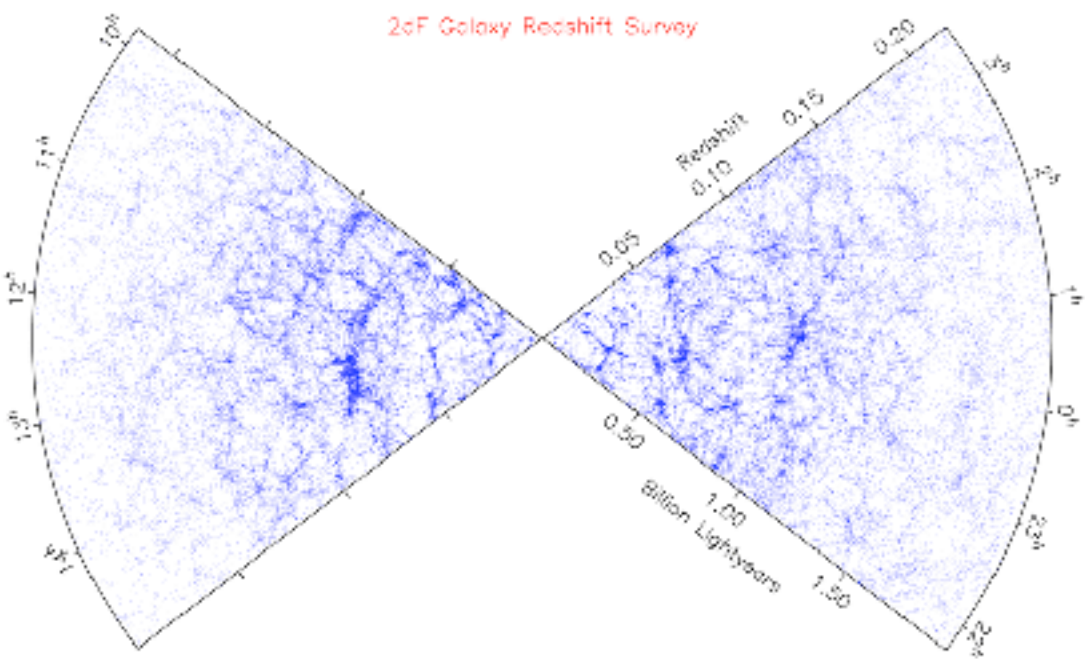
- The Linear Universe:

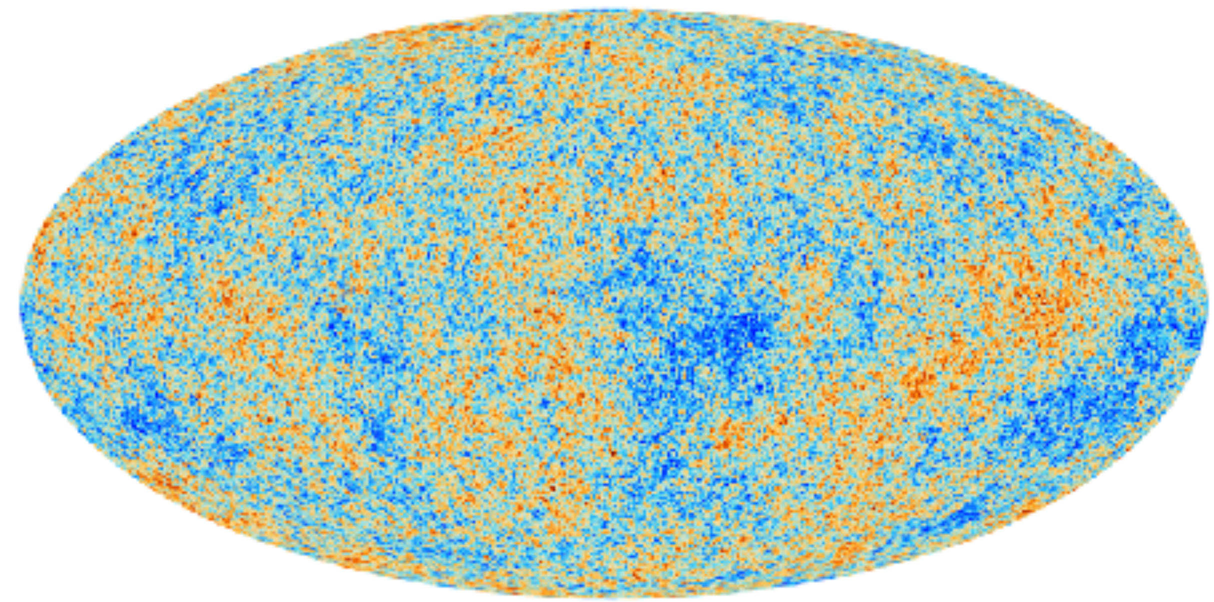
Characterize statistics of inhomogeneities!

$$\mathcal{P}\left[g_{\mu\nu}\left(x, t=0\right)\right] \quad \mathcal{P}\left[\rho_i\left(x, t=0\right)\right] \quad \mathcal{P}\left[{u_i}_\mu\left(x, t=0\right)\right]$$

Assume our Universe is typical.

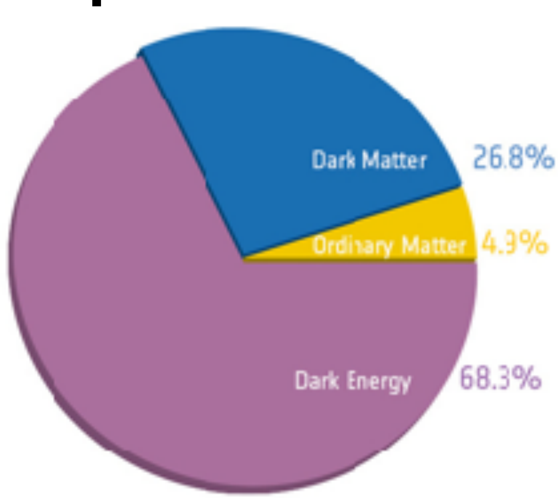
6 Parameter Model of the Universe



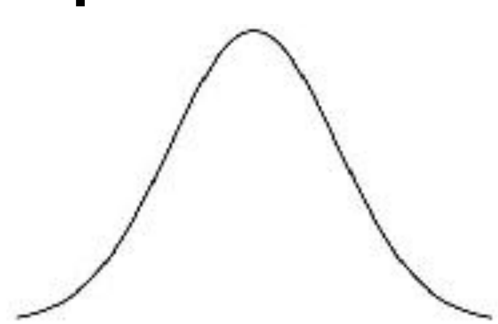


$$\Lambda\mathrm{CDM}$$

3 parameters



2 parameters



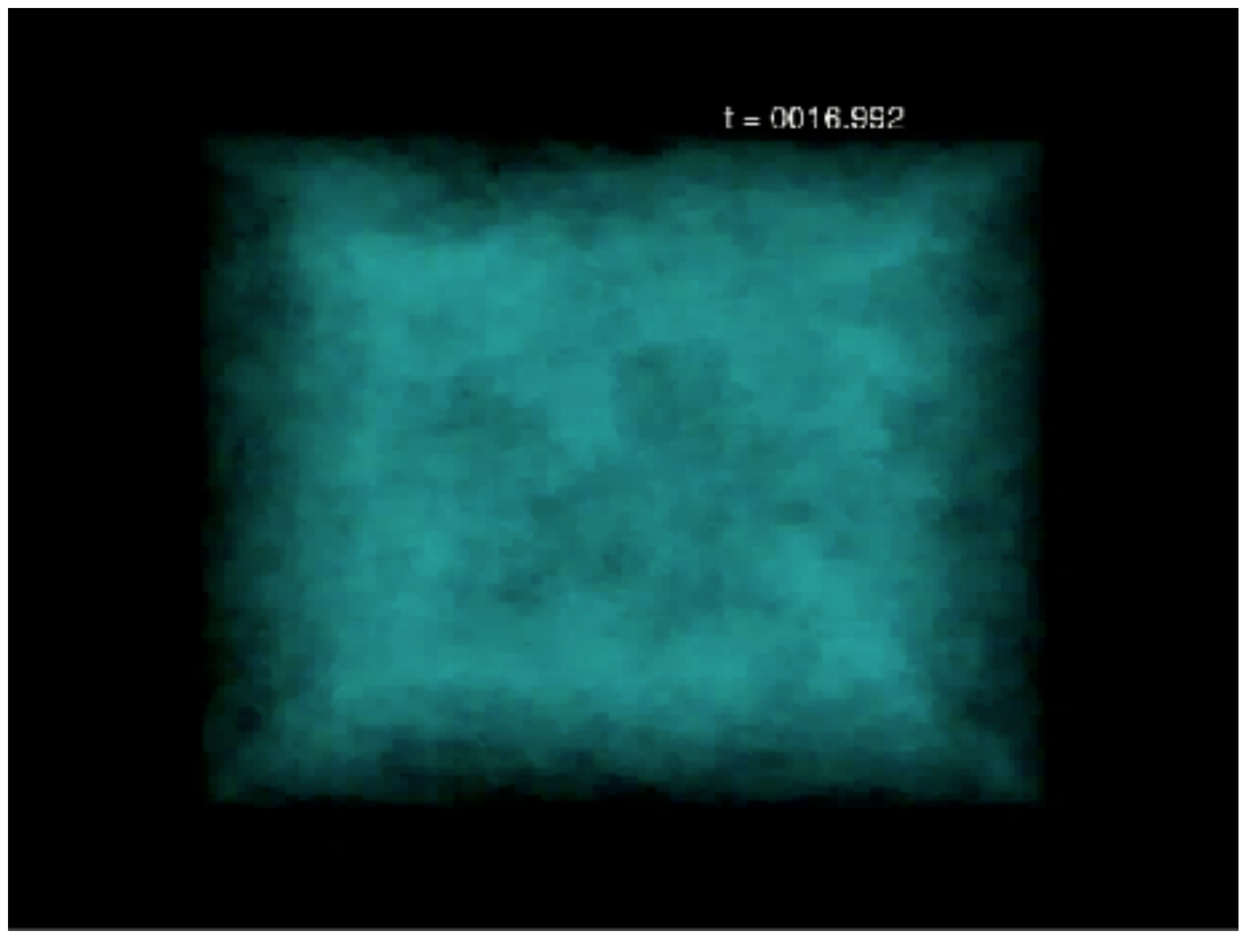
$$\mathcal{P}\left[\delta g_{\mu\nu}\left(x, t=0\right)\right]$$

1 parameter (for CMB)

$$\tau$$

Modelling the universe

Initially small fluctuations collapse to form galaxies, stars, etc.



That's it!

The rest is details.

Giving Thanks

- The non-linear Universe
  - GR is highly non-linear - inferring the state of the early universe would be like asking for the weather 100 million years ago based on the weather today.
  - No general classification of metrics - how to characterize initial conditions?
  - Shock waves, singularities, baryonic feedback, oh my!

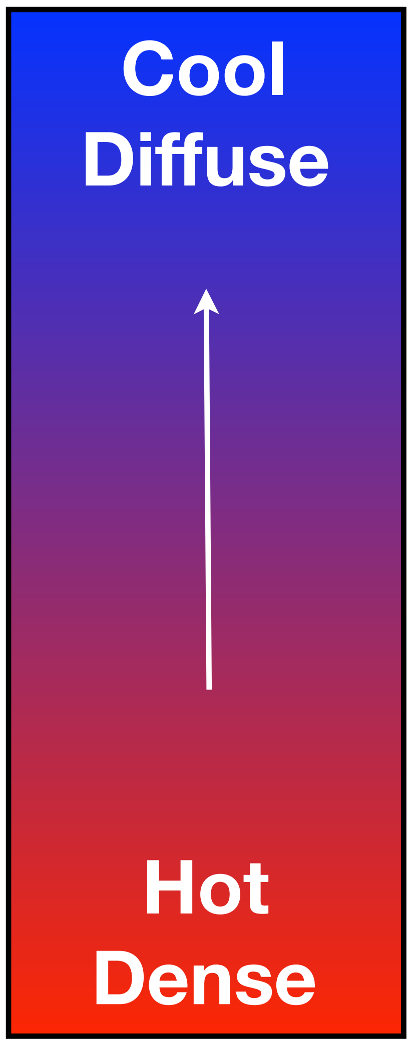
Giving Thanks

- The linear Universe
    - Simple evolution allows initial conditions to be inferred.
    - Background evolution and growth of structure can be analyzed separately.
    - Simple classification of initial conditions and metric degrees of freedom.
    - Physics on different scales evolves independently (Fourier modes independent).

The rest

Now for some details....

The homogeneous universe



13.7 Billion Years: the present.

9.1 Billion Years: our sun ignites.

100 million years galaxies and first stars form.

380,000 years: neutral atoms form.

1 second: atomic nuclei form.

10^{-6} seconds: protons and neutrons form.

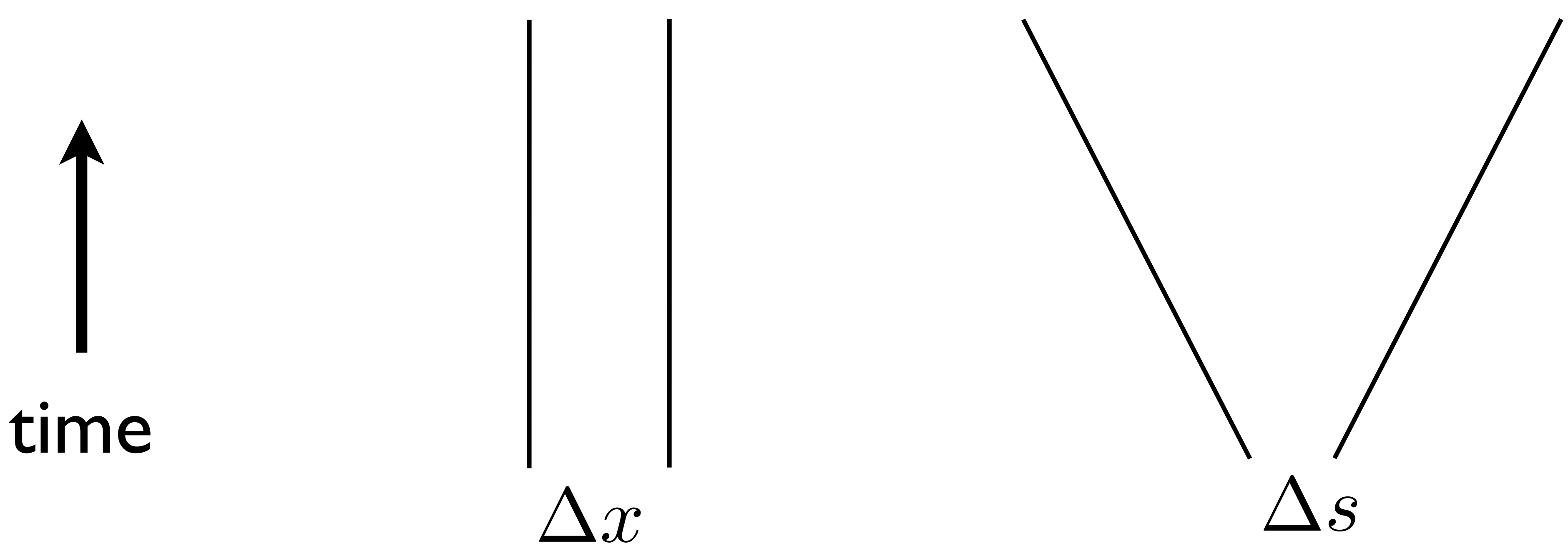
?Big Bang?

The homogeneous universe

- The metric in a flat, homogeneous, isotropic universe:

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

comoving coordinates

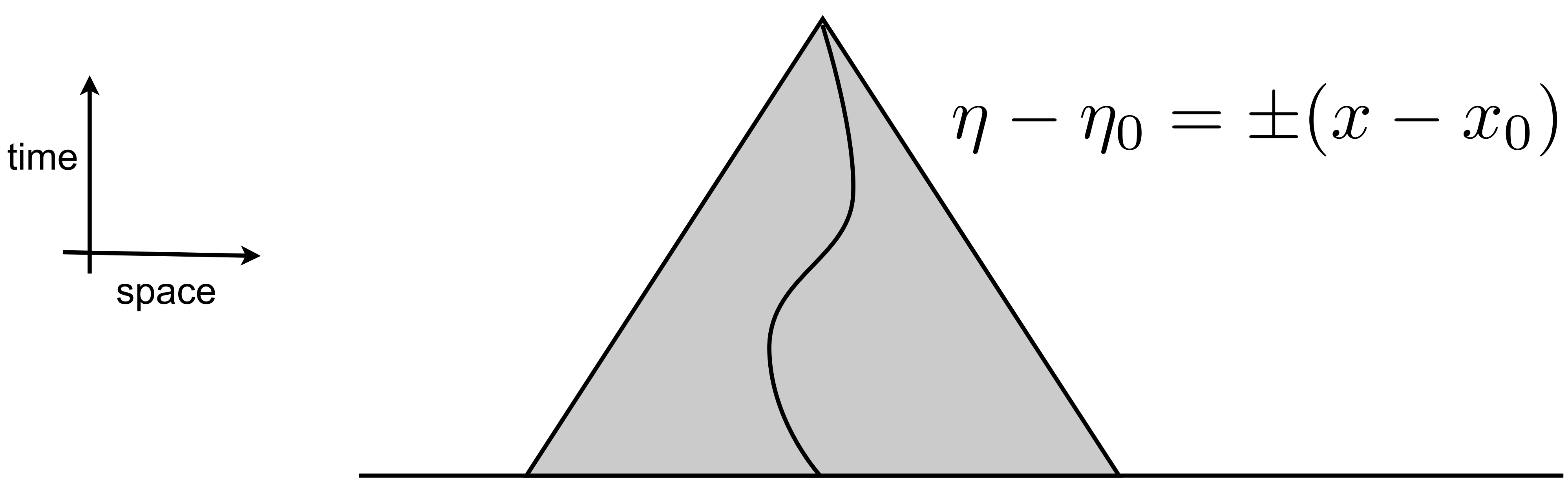


constant comoving distance = growing physical distance

The homogeneous universe

- Conformal time:

$$\eta = \int \frac{dt}{a(t)} \qquad\qquad ds^2 = a^2(\eta)\left[-d\eta^2 + \delta_{ij}dx^i dx^j\right]$$



The big bang: when a(t)=0

The homogeneous universe

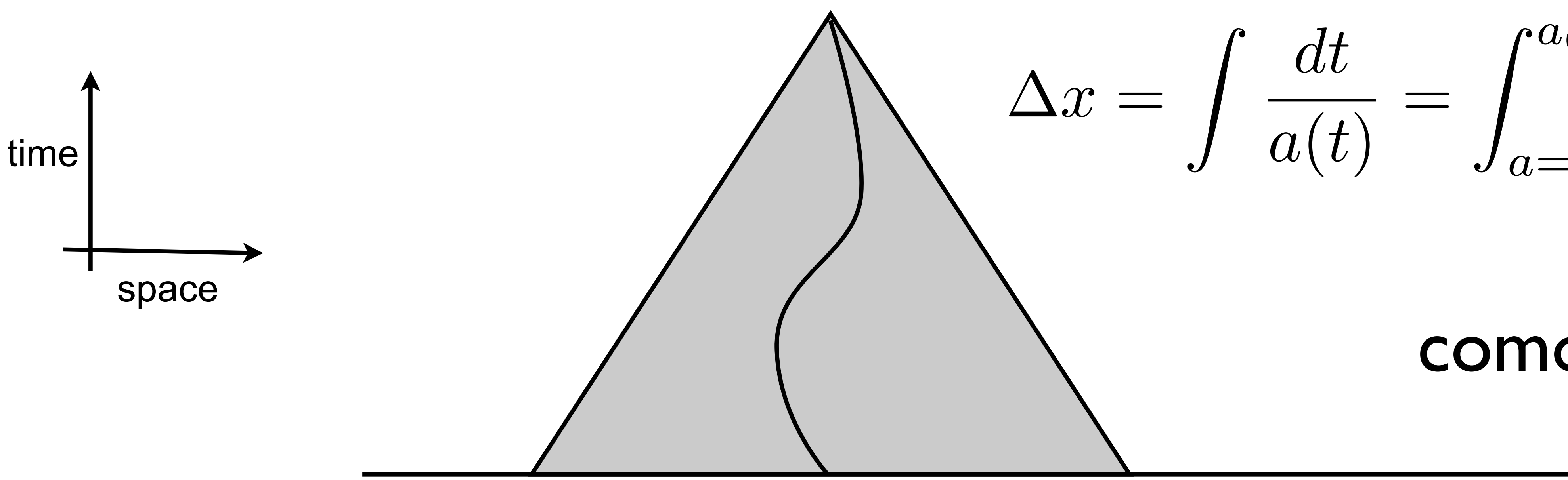
- Conformal time:

$$\eta = \int \frac{dt}{a(t)} \qquad\qquad ds^2 = a^2(\eta)\left[-d\eta^2 + \delta_{ij}dx^i dx^j\right]$$

particle horizon

$$\Delta x = \int \frac{dt}{a(t)} = \int_{a=0}^{a_0=1} \frac{d\ln(a)}{aH}$$

comoving horizon



The big bang: when a(t)=0

The homogeneous universe

- Equations of motion in a homogeneous universe:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \Longrightarrow \quad H^2 \equiv \left(\frac{\dot{a}}{a}\right) = \frac{8\pi G\rho}{3}$$

$$\nabla_\mu T^{\mu\nu} = 0 \quad \Longrightarrow \quad \dot{\rho} = -3H\left(\rho + p\right)$$

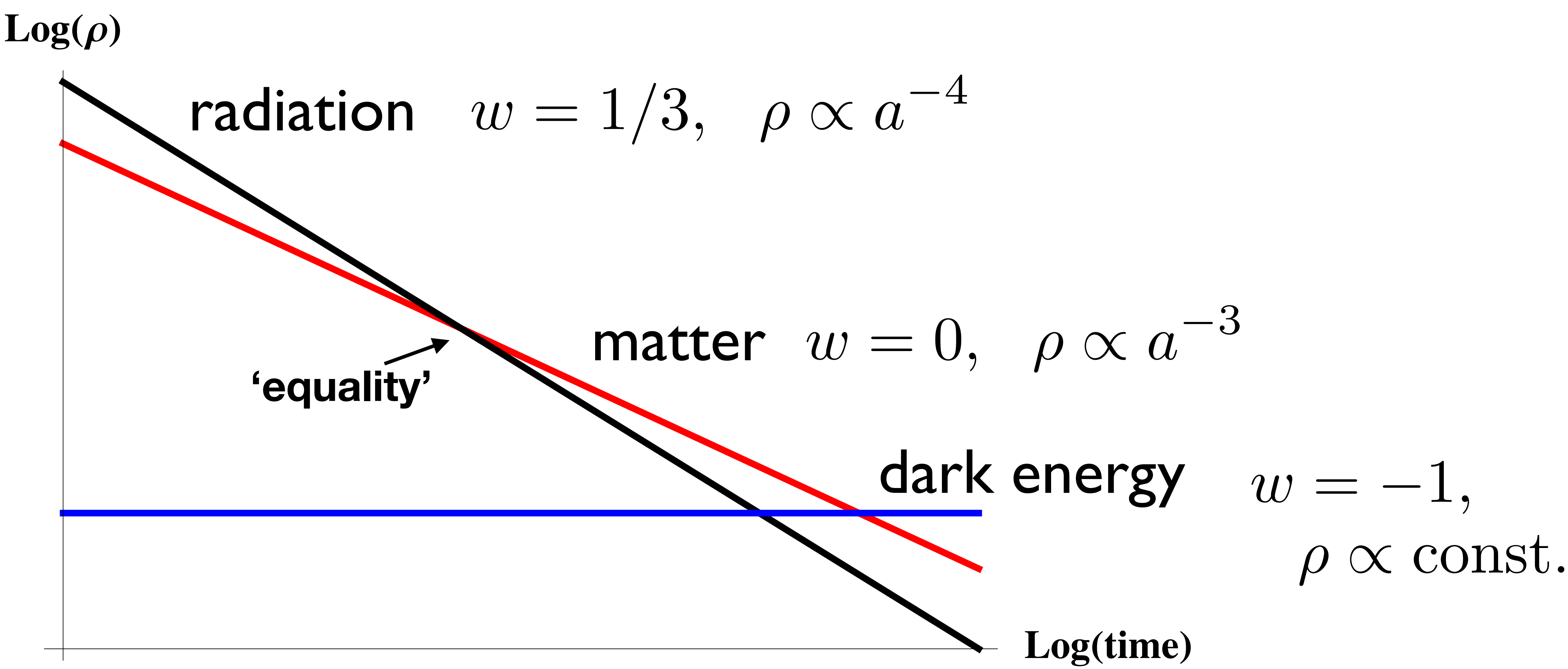
$$p = w\rho$$

- Solutions:

$$\rho = \rho_0 a^{-3(1+w)} \qquad a(t) = a_0 t^{\frac{2}{3(1+w)}}$$

different fluids gravitate differently!

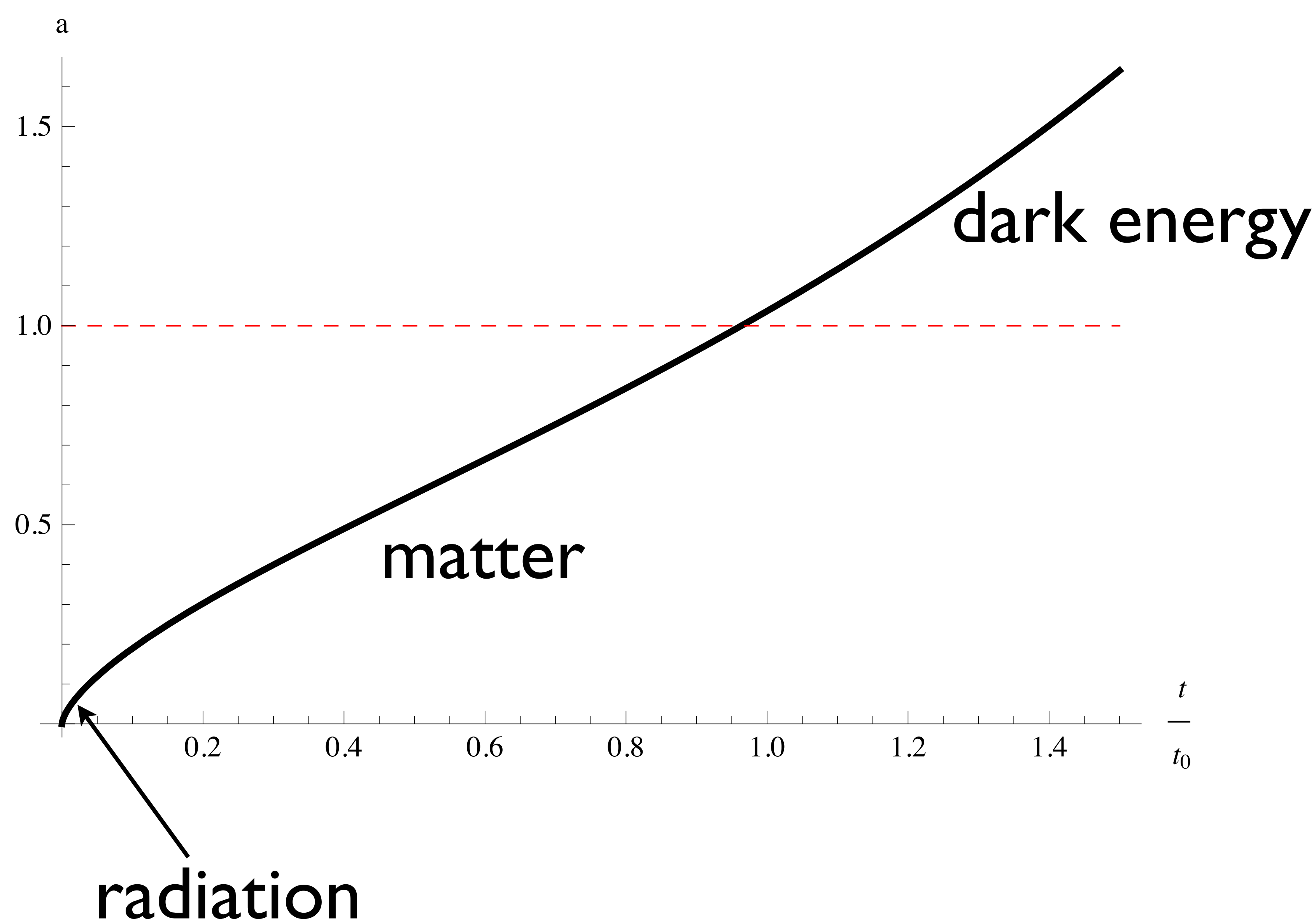
The homogeneous universe



$$\rho = \rho_0 a^{-3(1+w)}$$

The homogeneous universe

- Evolution of the scale factor:



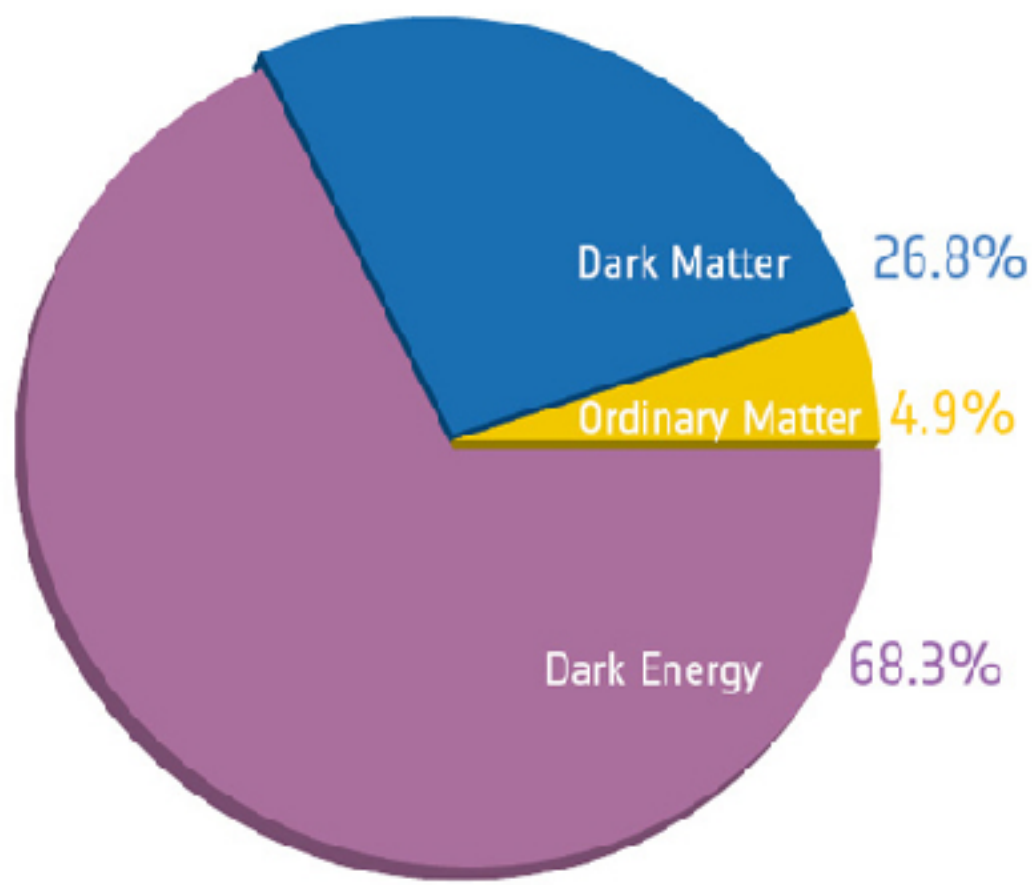
The homogeneous universe

- Energy budget:

$$\left(\frac{H}{H_0}\right)^2 = \sum_i \Omega_i a^{-3(1+w_i)} \qquad \sum_i \Omega_i = 1$$

$$H_0 = 67.4 \ \frac{\text{km}}{\text{s Mpc}}$$

$$c/H_0 = 4450 \ \text{Mpc}$$



$$(\Omega_r \sim 10^{-4})$$

Gravitational Instability

- For non-relativistic matter in flat space:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

continuity

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\, \vec{v} + \frac{\nabla p}{\rho} + \nabla \phi = 0$$

Euler

$$\nabla^2 \phi = 4\pi G \rho$$

Poisson

- Linearize:

$$\rho = \bar{\rho} + \delta\rho \qquad \phi = \bar{\phi} + \delta\phi \qquad \vec{v} = \bar{\vec{v}} + \delta\vec{v}$$

$$p = \bar{p} + c_s^2 \delta\rho$$

$$c_s^2 = \frac{\partial p}{\partial \rho} = w$$

Gravitational Instability

- Linearized equation of motion:

$$\frac{\partial^2 \delta\rho}{\partial t^2} - c_s^2 \nabla^2 \delta\rho - 4\pi G \bar{\rho} \delta\rho = 0$$

- Fourier transform:

$$\delta\rho(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \delta\rho(t, k) e^{i\vec{k}\cdot\vec{x}} \qquad \frac{\partial^2 \delta\rho}{\partial t^2} + \left(c_s^2 k^2 - 4\pi G \bar{\rho}\right) \delta\rho = 0$$

$$\delta\rho(t, k) = A \exp\left(i\omega(k)t\right) + B \exp\left(-i\omega(k)t\right)$$

$$\omega(k) = \sqrt{k^2 c_s^2 - 4\pi G \bar{\rho}}$$

Gravitational Instability

$$\omega(k) = \sqrt{k^2 c_s^2 - 4\pi G\bar{\rho}}$$

- Jeans scale: competition between pressure and gravity

$$\lambda_J = \frac{2\pi}{k_J} = c_s \left(\frac{\pi}{G\bar{\rho}}\right)^{1/2}$$

Gravitational Instability

$$\omega(k) = \sqrt{k^2 c_s^2 - 4\pi G\bar{\rho}}$$

- Jeans scale: competition between pressure and gravity

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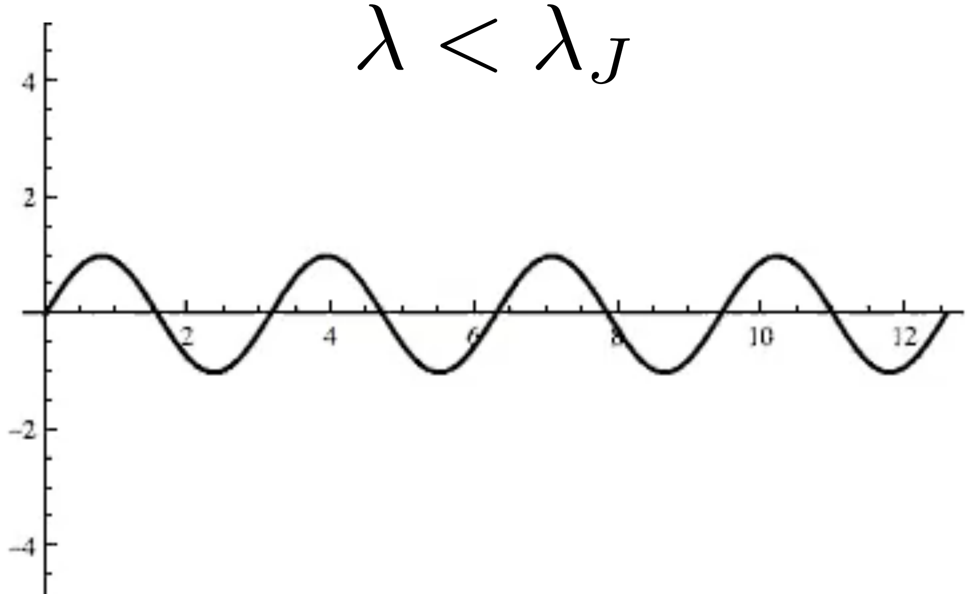
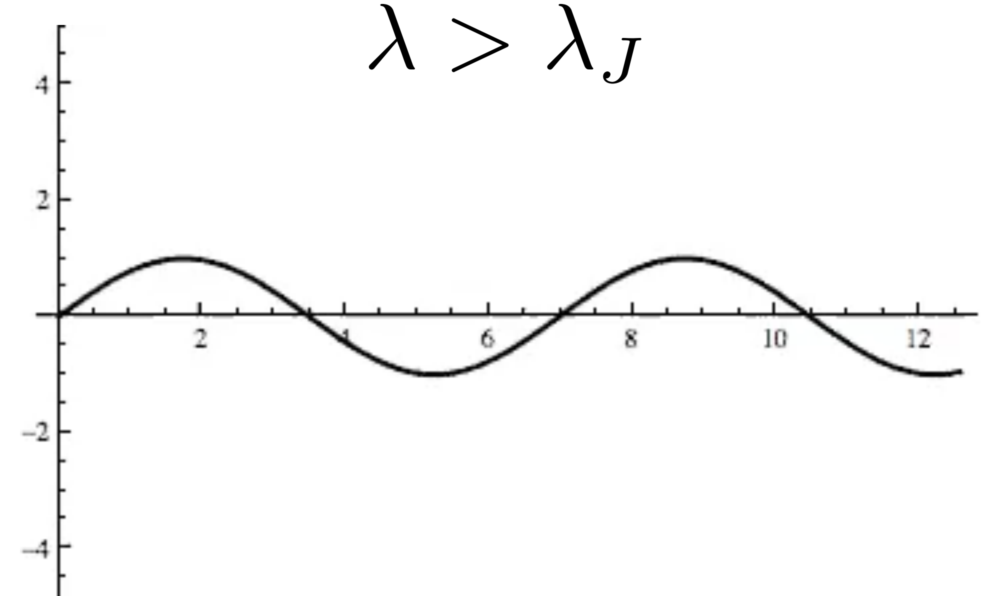
$\lambda < \lambda_J$



sound waves

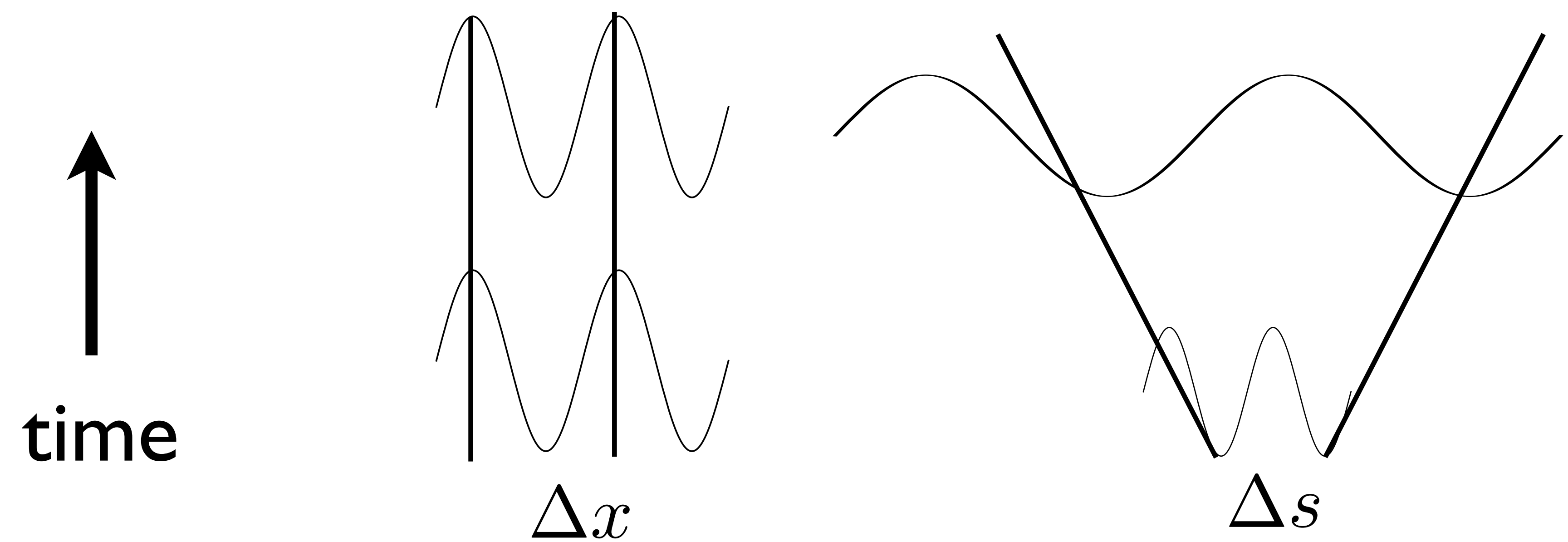
$\lambda > \lambda_J$



gravitational collapse

Gravitational Instability

- In an expanding universe waves are stretched:



$$\lambda_{\rm ph} = a(t)\lambda_{\rm com}$$

- Only gravitationally bound (non-linear) structures separate from the Hubble flow.
- Expansion inhibits collapse.

Gravitational Instability

$t_{\mathrm{coll}} \sim (4\pi G\bar{\rho})^{-1/2} \sim H^{-1}$ Expansion will be relevant!

- Including expansion (on small scales):

$$\left[\frac{d^2}{dt^2} + 2H\frac{d}{dt} + \left(c_s^2\frac{k_{\mathrm{com}}^2}{a^2} - 4\pi G\bar{\rho}\right)\right]\frac{\delta\rho(t, k_{\mathrm{com}})}{\bar{\rho}(t)} = 0$$

$c_s^2 = 0$

radiation	matter	dark energy
$\frac{\delta\rho}{\bar{\rho}} \propto \log(a)$	$\frac{\delta\rho}{\bar{\rho}} \propto a$	$\frac{\delta\rho}{\bar{\rho}} \propto \mathrm{const.}$

Growth

- On very large scales, there is no growth — need GR.

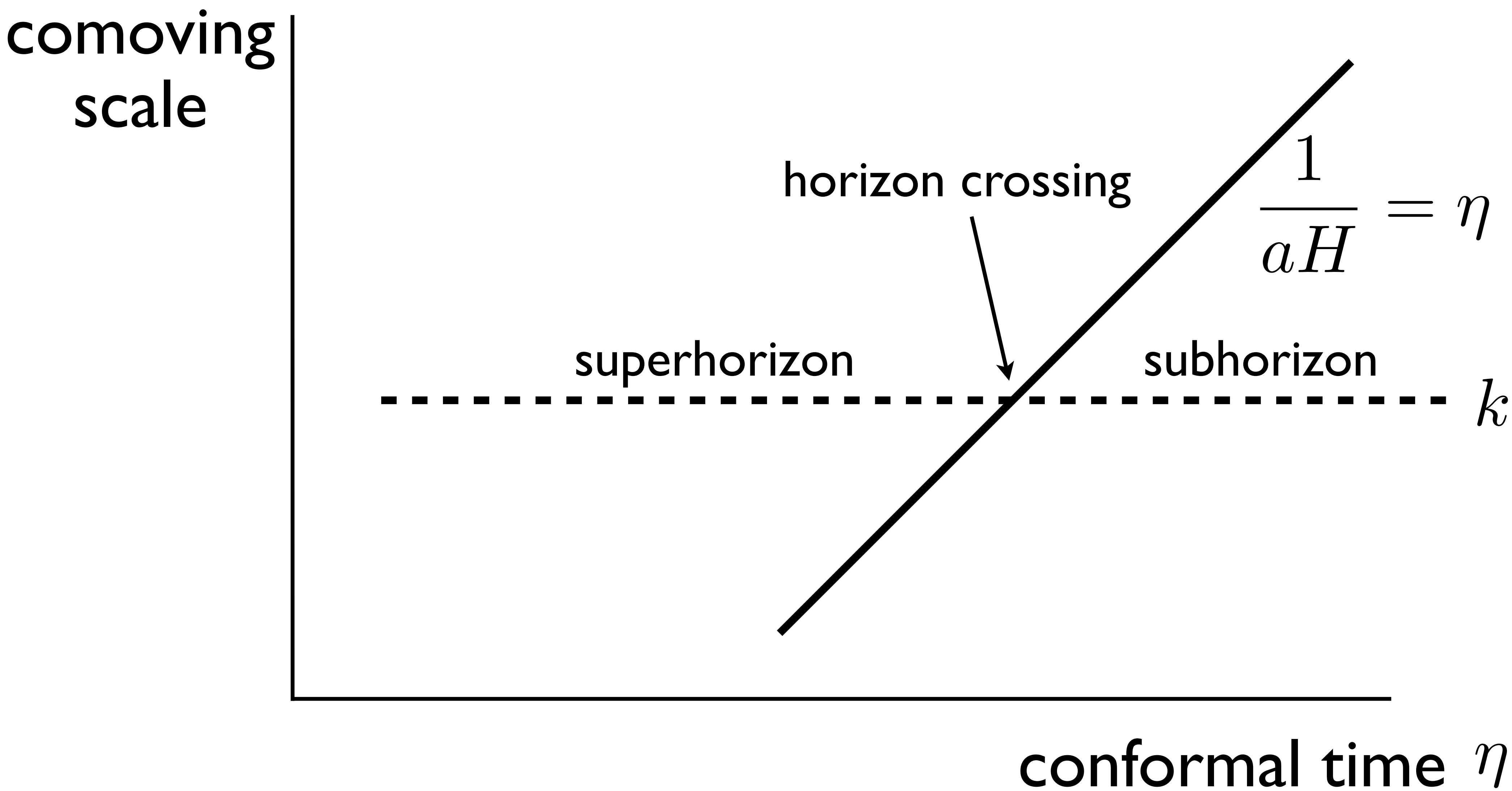
- Between $t_{\rm eq} < t < t_0$, on small scales, density can grow by a factor of about $1/a_{\rm eq} \sim 3000$

To have structure, need: $$\frac{\delta\rho}{\bar{\rho}}(t_{\rm eq}) > a_{\rm eq} = \frac{\Omega_r}{\Omega_m}$$

- If there were no dark matter, matter-radiation equality happens later and growth during matter domination would be a factor of 5 smaller!

The most important plot

- An important scale: comoving horizon $\frac{1}{aH}$
- Horizon crossing: $k = aH$



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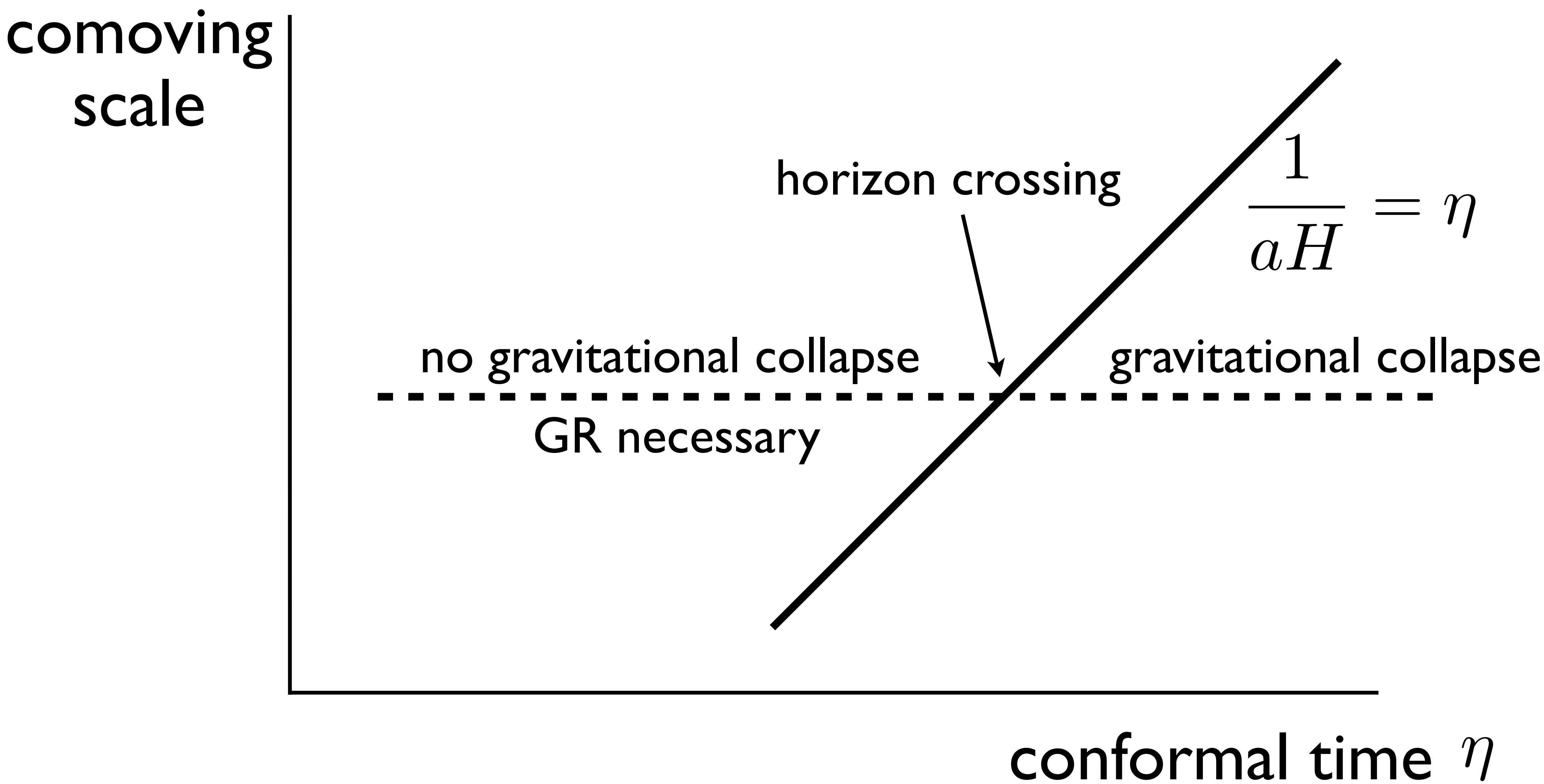

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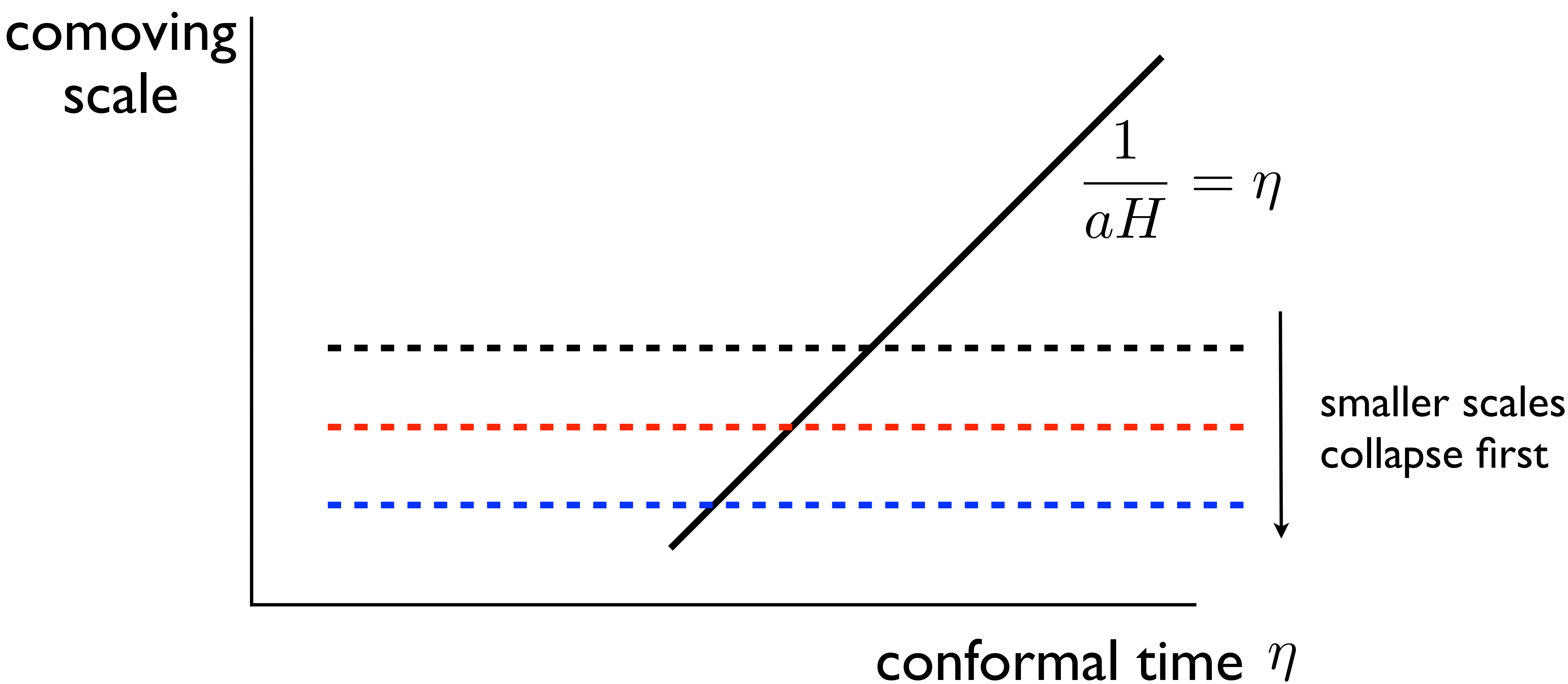

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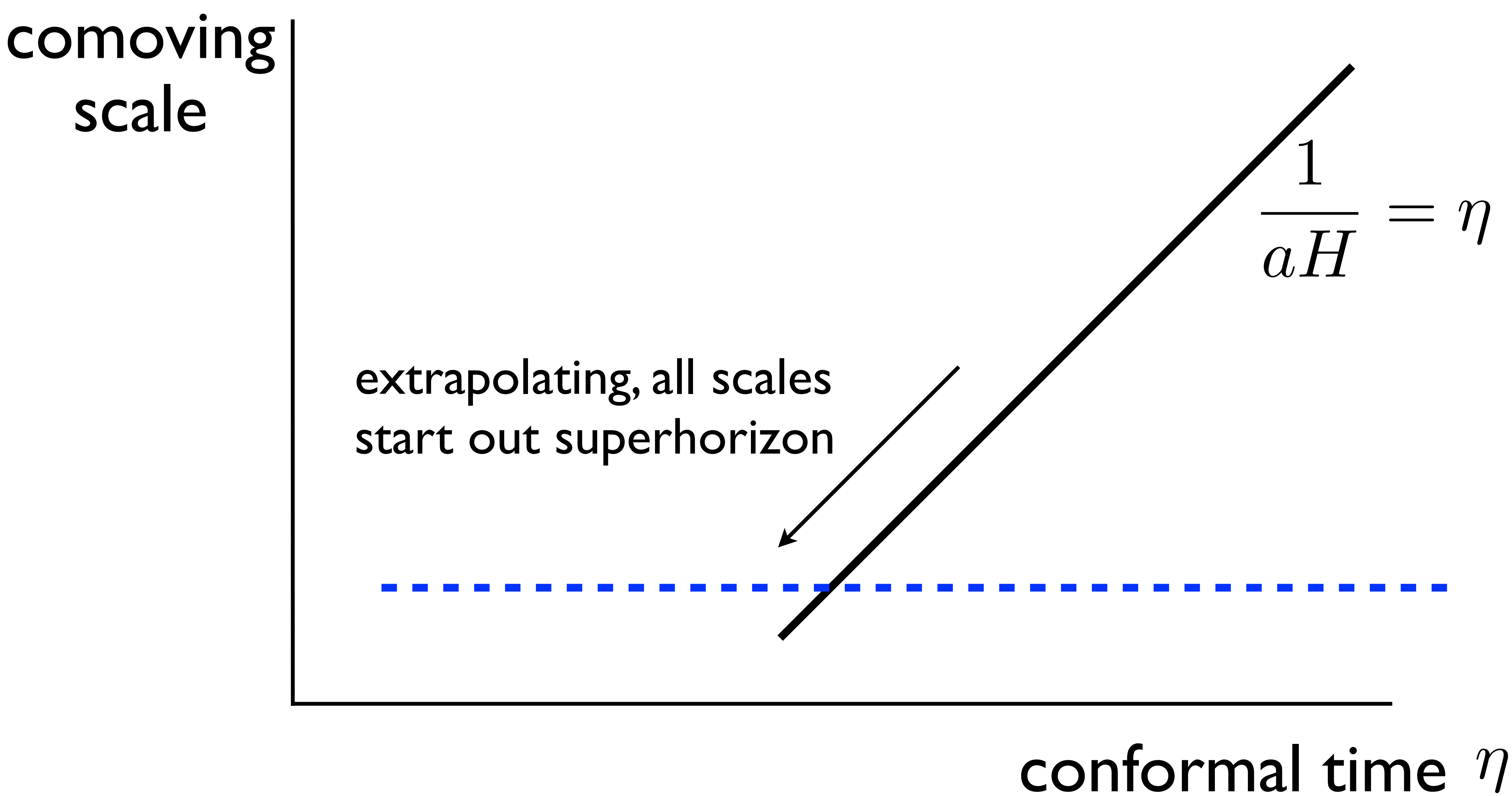

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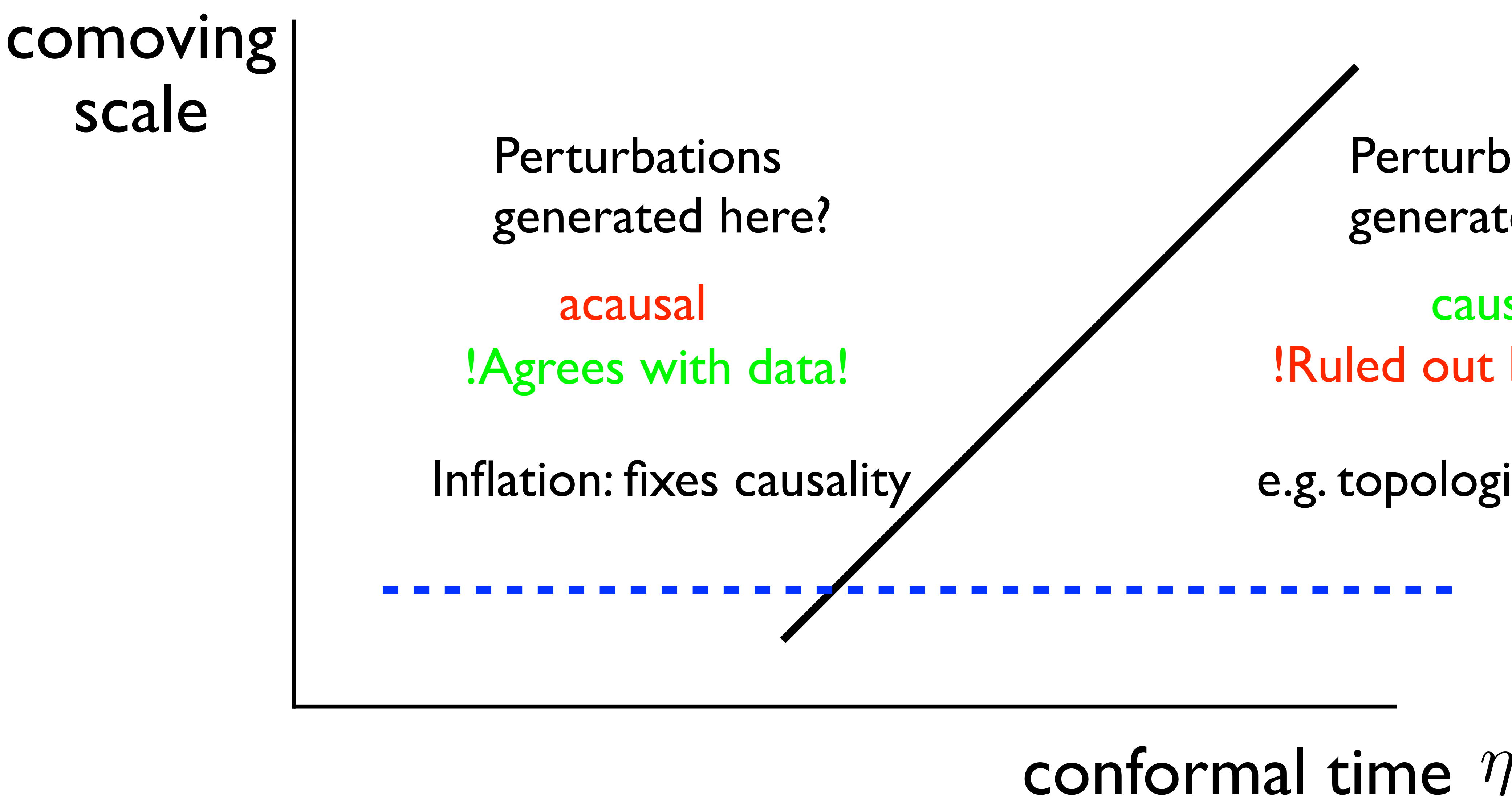

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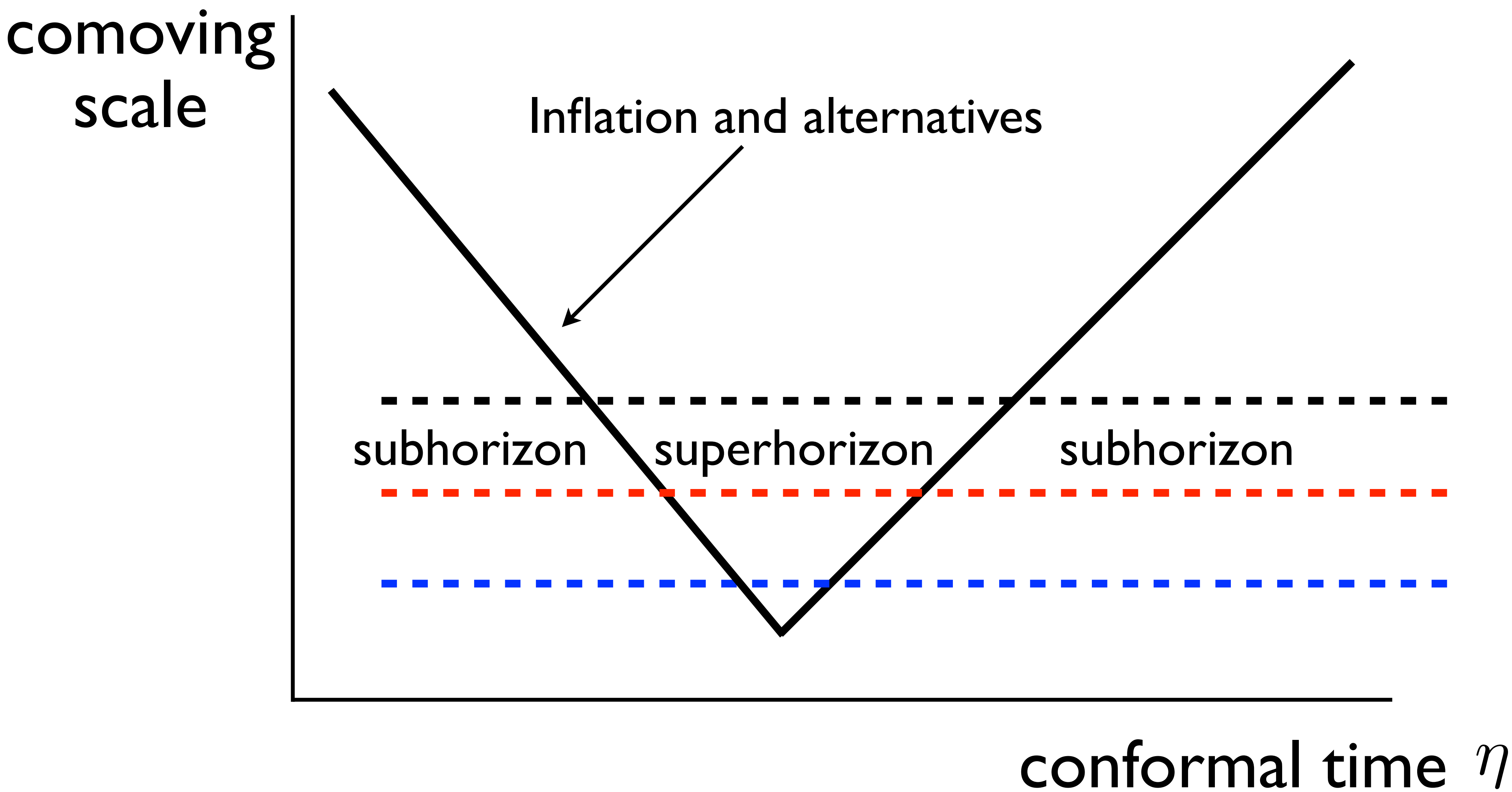
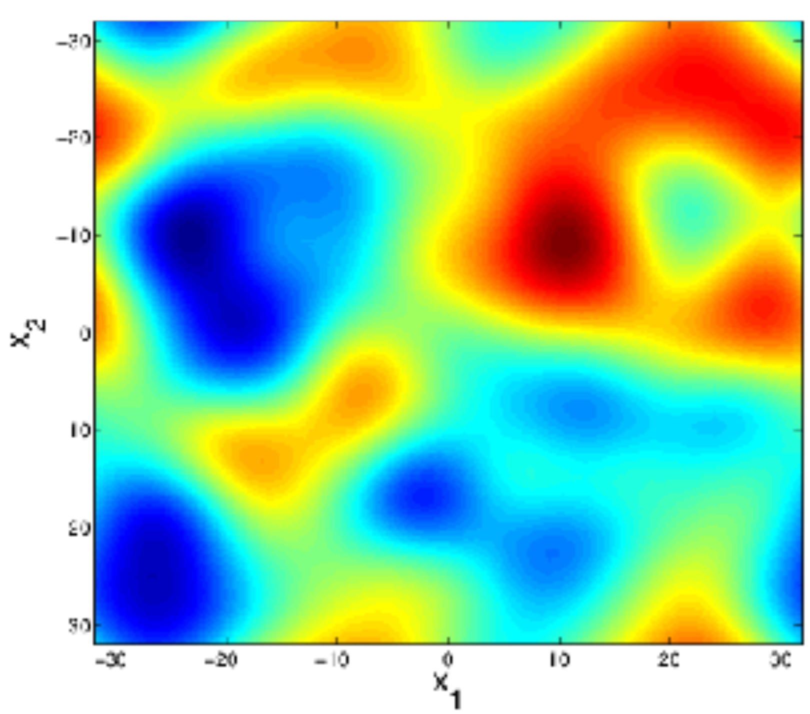


Initial Conditions

- The initial fluctuations are imprinted in the gravitational potential.
- Empirically, the potential is a random Gaussian field whose power spectrum (all the info!) is nearly scale invariant (equal power in each decade of scale):

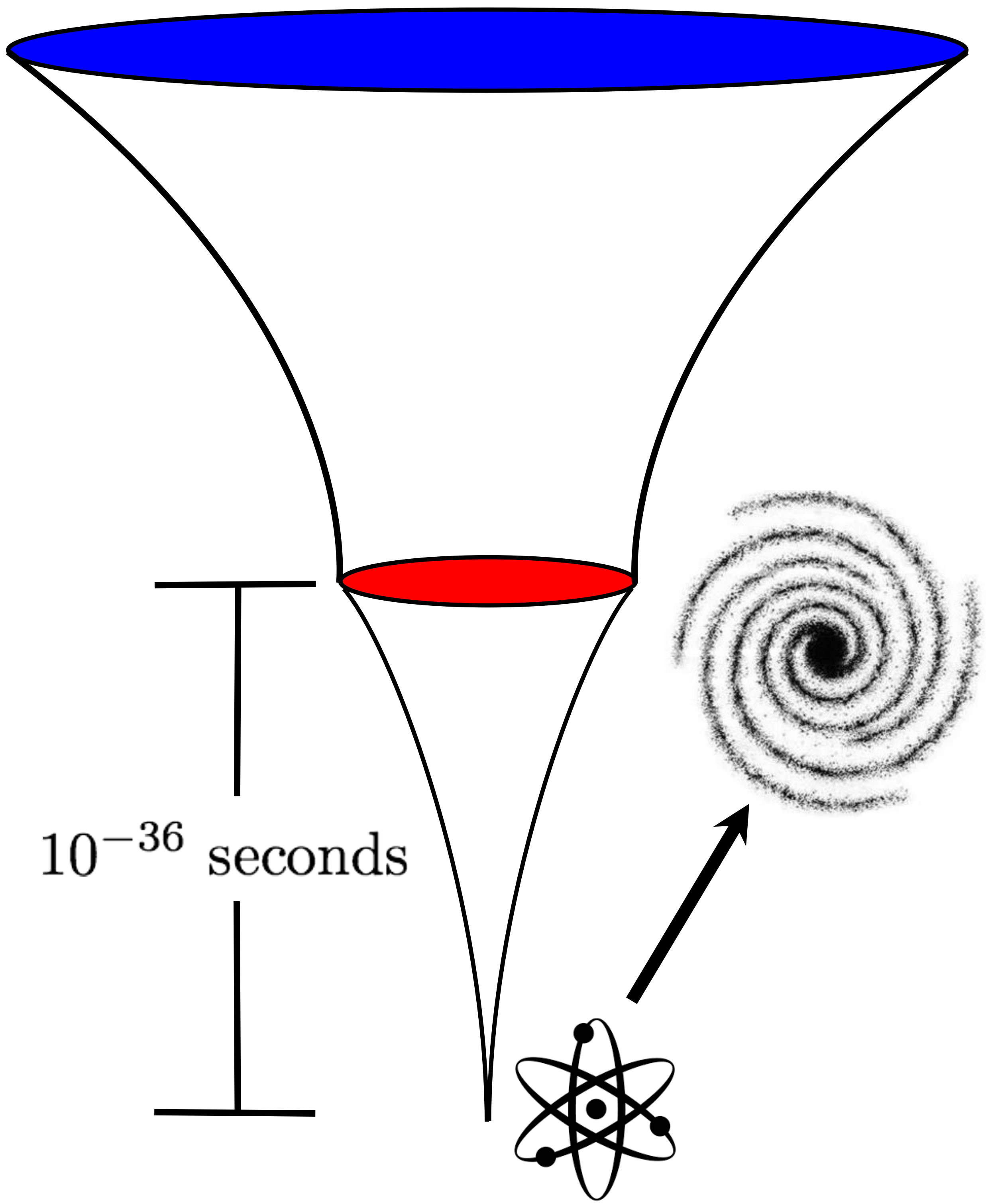


$$\langle \phi(\vec{k})\phi(\vec{k}')\rangle = \delta^{(3)}(\vec{k}-\vec{k}')P_\phi(k)$$

$$P_\phi(k) = \frac{A_s}{k^3}\left(\frac{k}{k_*}\right)^{n_s-1}$$

Adiabatic: density perturbations follow potential perturbations.

Inflation: the prequel to the Observable Universe

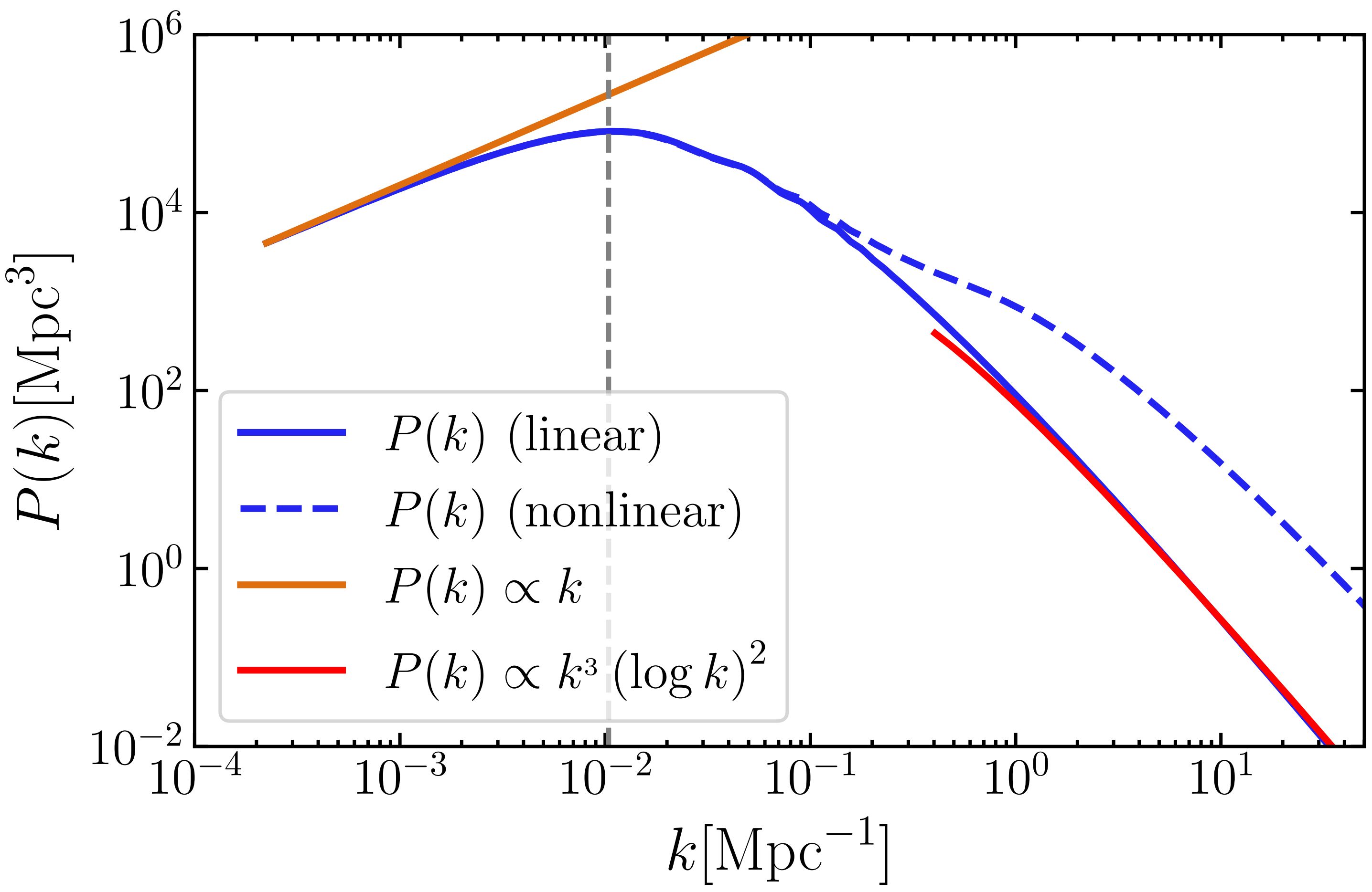


Matter Power Spectrum

- Relating the potential fluctuations to density perturbations:

$$P_m(k) = \frac{9}{25}\frac{k^4}{\Omega_m^2 H_0^4} T(k)^2 D(a)^2 P_\phi(k)$$

Transfer function Growth function

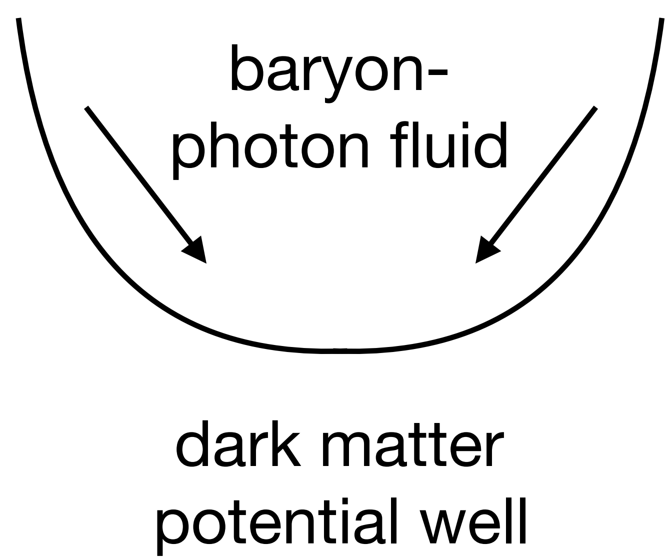


Baryons and photons

equality	recombination	reionization
photons tightly coupled to baryons imperfect fluid with $c_s^2 \simeq 1/2$	neutral atoms form	first non-linear structure
	photons and baryons decouple: CMB is released!	stars reionize the Universe
dark matter perturbations begin to grow	baryons begin to collapse into dark matter halos	hierarchical structure formation
		some CMB photons re-scatter

Baryon Acoustic Oscillations

- Before recombination, photons and baryons are coupled — pressure!



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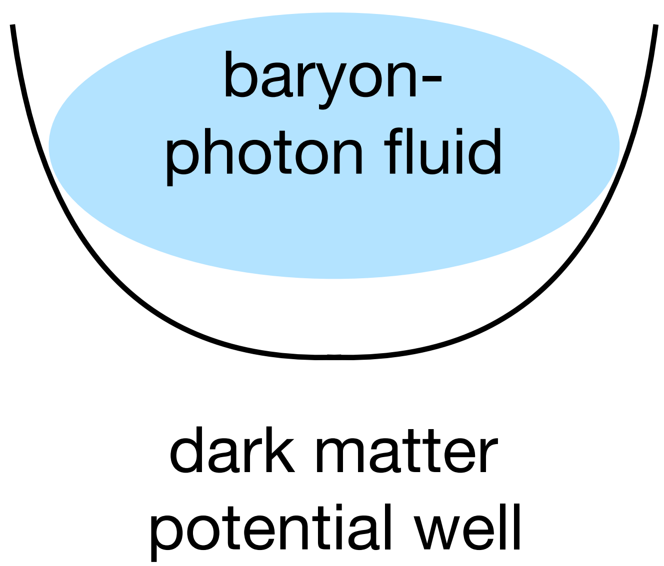

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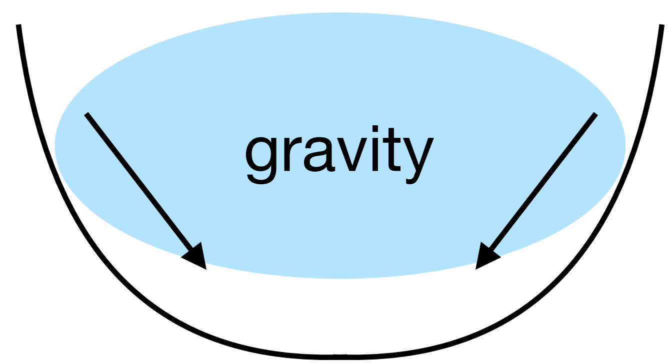
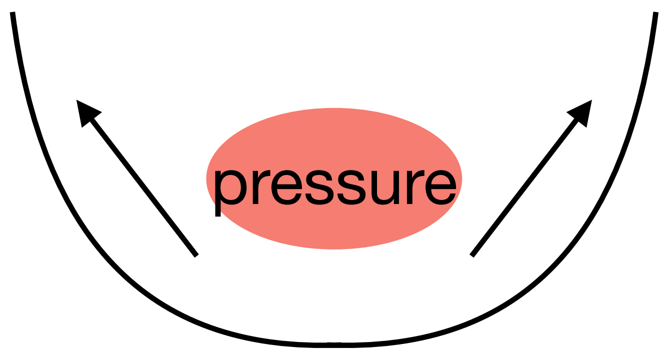


Baryon Acoustic Oscillations

- Before recombination, photons and baryons are coupled — pressure!



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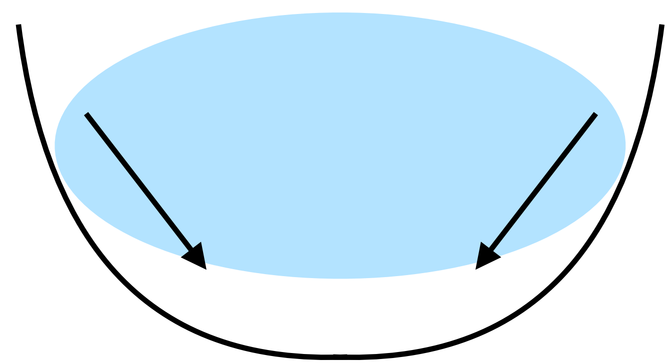
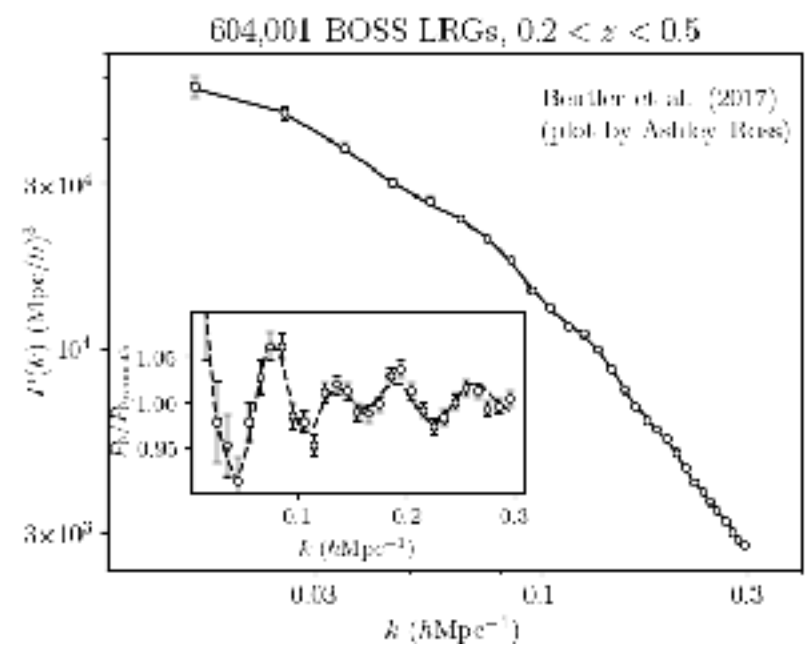


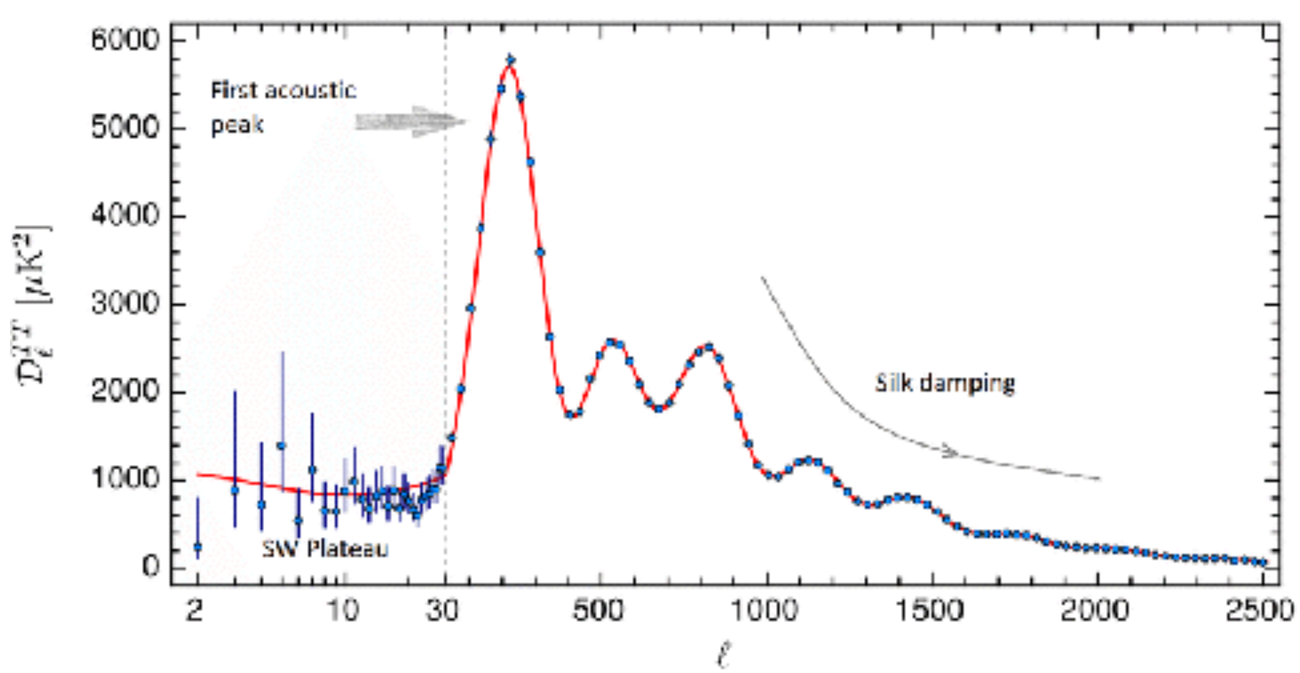
- After recombination, baryons decouple — scale of the sound horizon (distance sound travels since the big bang) is imprinted!

Baryon Acoustic Oscillations

- Matter power spectrum:



- Imprints in the distribution of photons (CMB):



Cold dark matter

- The model for linear growth of inhomogeneities described above assumed dark matter is composed of non-relativistic, non-interacting particles.

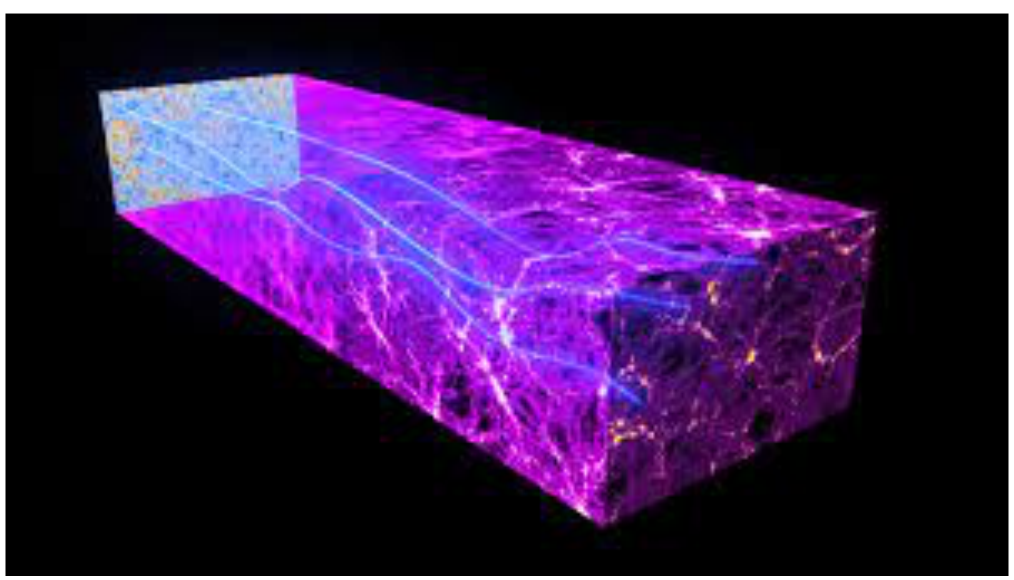
$$c_s = 0 \quad \longleftrightarrow \quad \textbf{cold dark matter (CDM)}$$

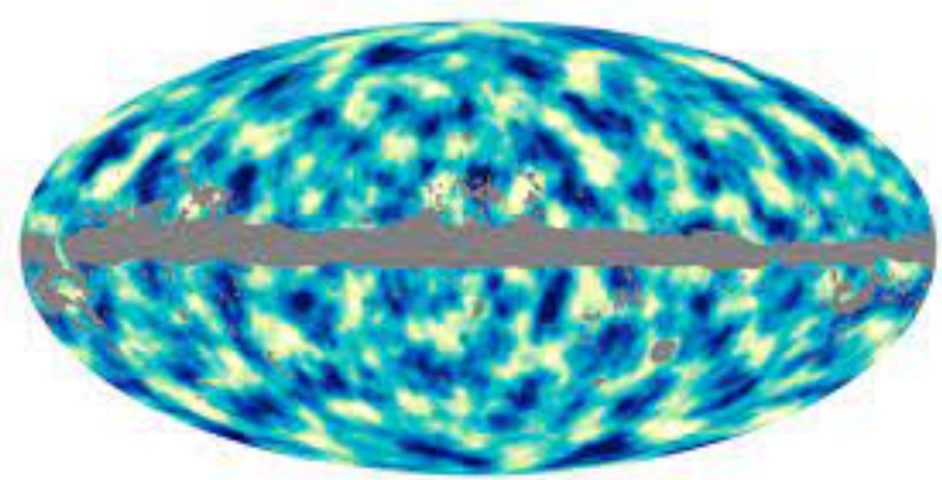
- Massless dark matter particles would be 'hot' and behave like radiation — pressure! Not much growth and no acoustic oscillations, power suppression on scales beyond the peak of the matter power spectrum.

- Warm dark matter — dark matter particles with small mass become non-relativistic some time during cosmic history. Power suppression on small scales, can preserve BAO.

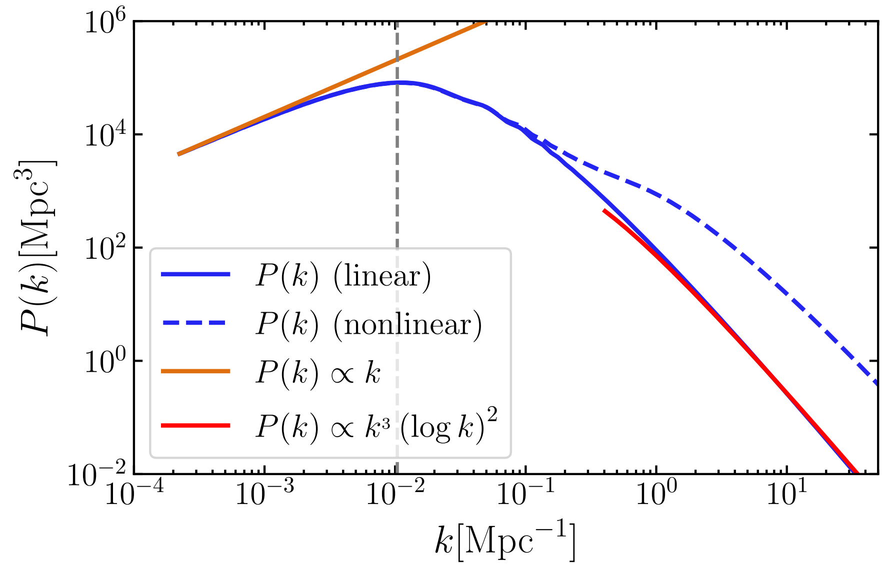
Dark Matter

- Cosmology provides ample evidence for dark matter:
  - Luminous matter not enough to account for growth.
  - BAO peaks in matter power spectrum and CMB angular power spectrum require dark matter.
  - Much more! For example, weak lensing of CMB.





Nonlinear scales

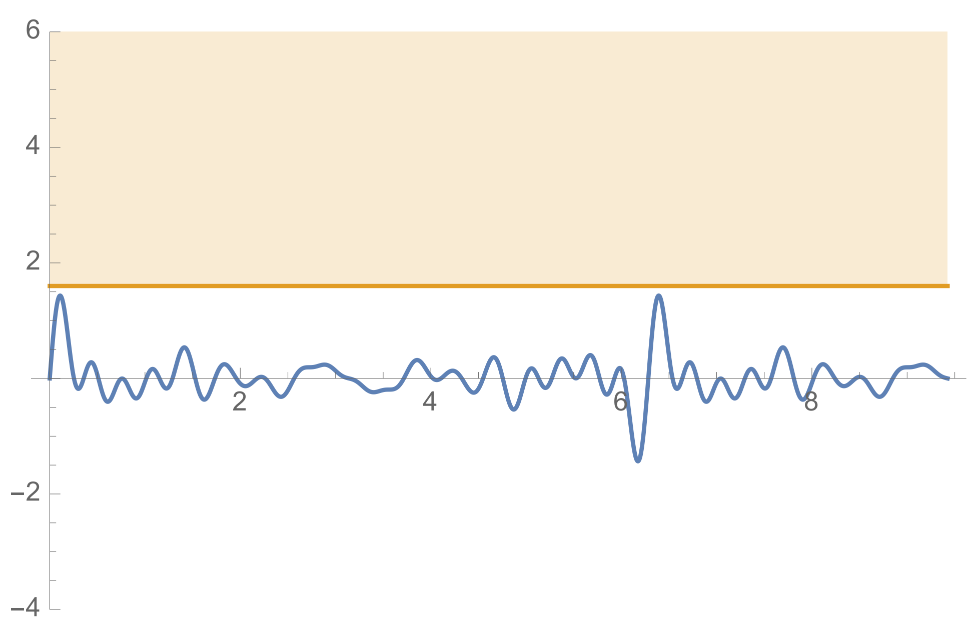


- Inhomogeneities smaller than ~ 50-100 Mpc are non-linear.

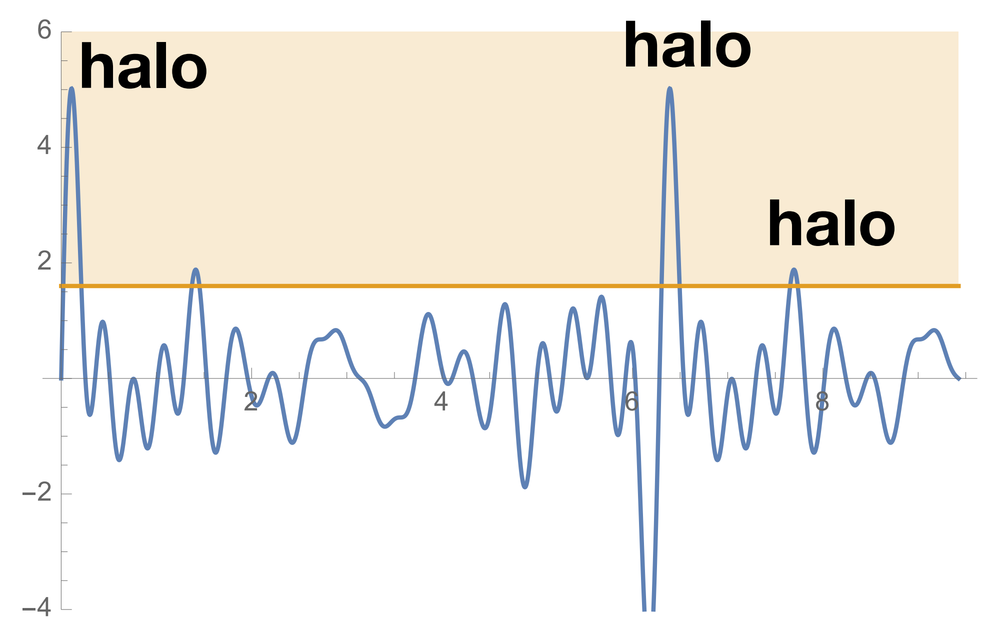
- These are the 'structures' that fill the Universe; in fact, it is not a bad approximation to assume that all dark matter is bound up into halos of varying mass.

Formation of dark matter halos

- Primordial dark matter perturbations are a random Gaussian field.
- Growth will drive peaks in the distribution into the non-linear regime — gravitational collapse happens in these regions.



Early time



Later time

Formation of dark matter halos

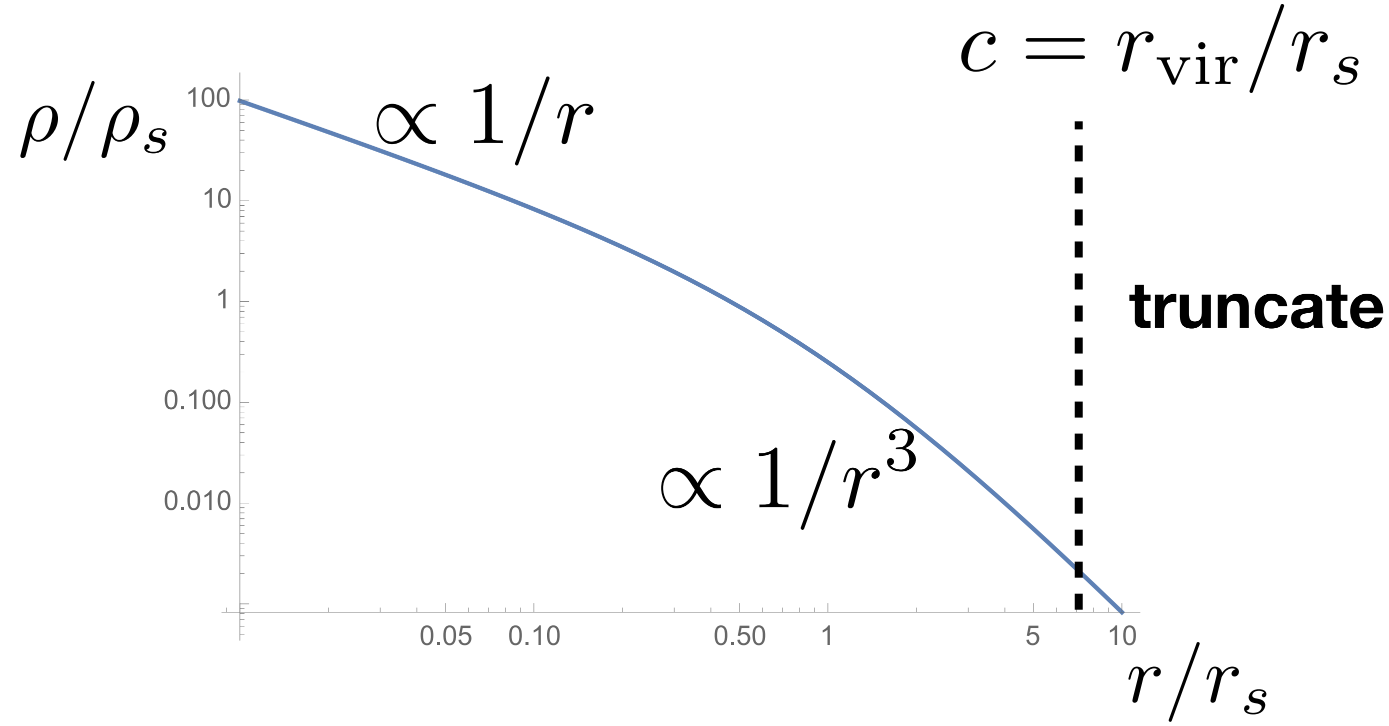
- Three stages of collapse:
  - Linear collapse: ~linear growth of overdensity to critical value of $\delta\rho/\bar{\rho} \simeq 1.686$.
  - Virialization: non-linear evolution to a virialized halo.
  - Secondary infall: surrounding dark matter is accreted.

NFW Profile

- The result: NFW profile

$$\rho = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$$

$$m = 4\pi\rho_s r_s^3 \left[\ln(1+c) - \frac{c}{1+c}\right]$$

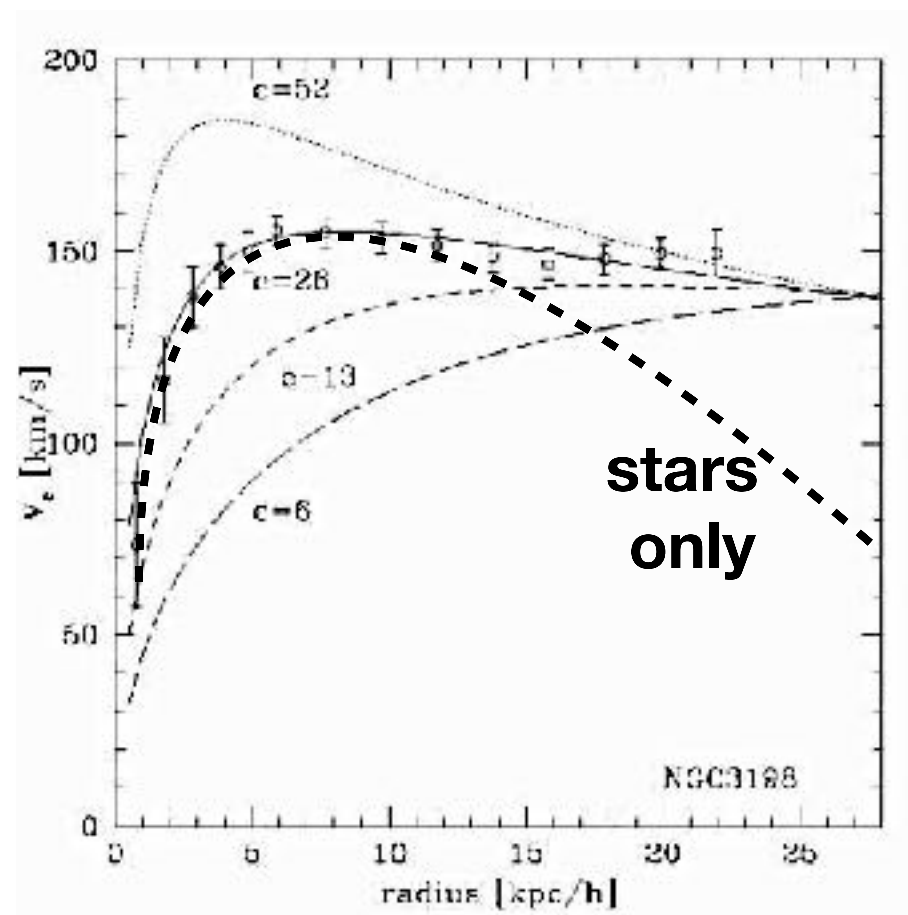


- A reasonable fit to N-body simulations.

Rotation Curves

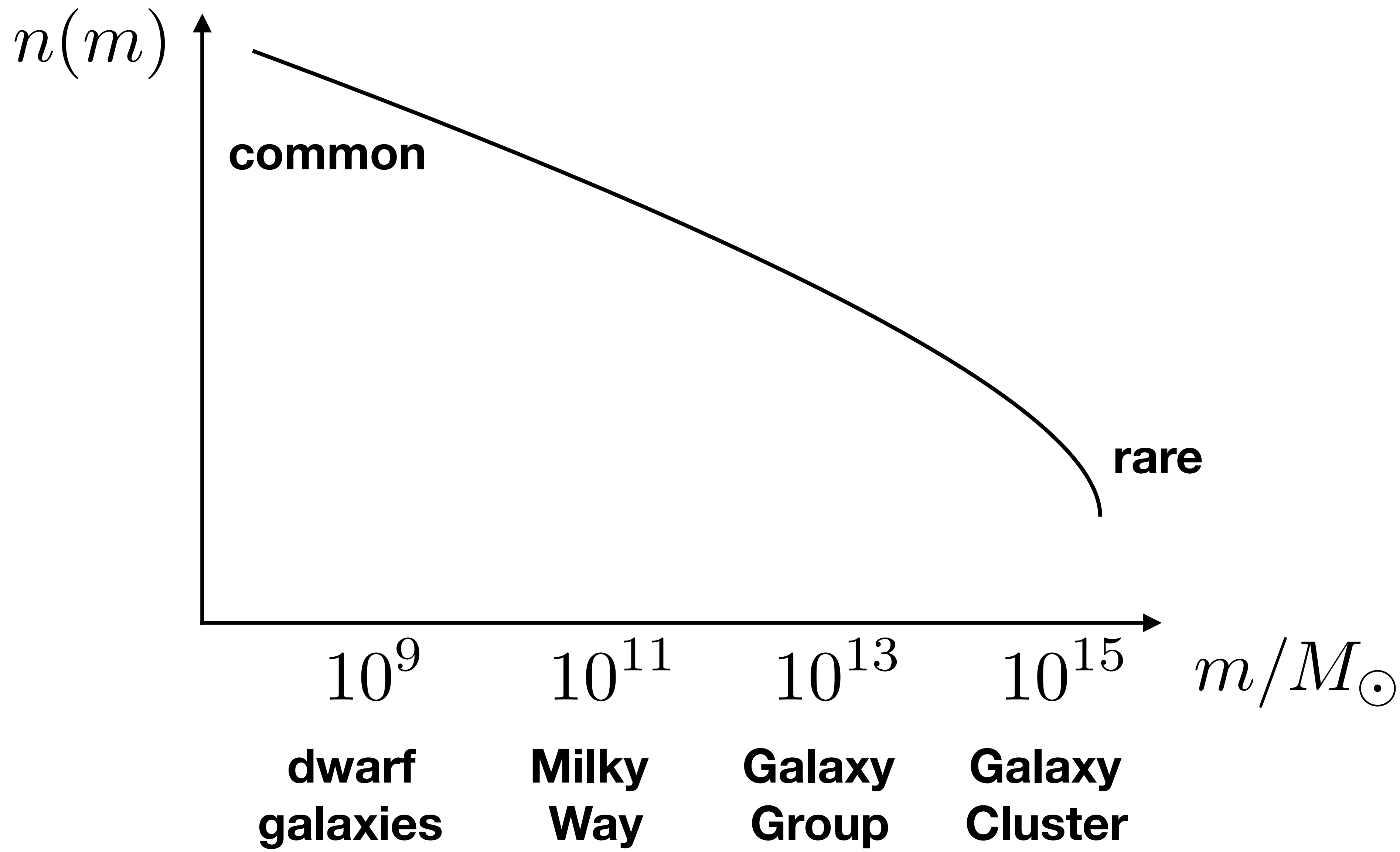
- Circular orbital velocity:

$$V_c(R) = \sqrt{\frac{G_N m(R)}{R}}$$



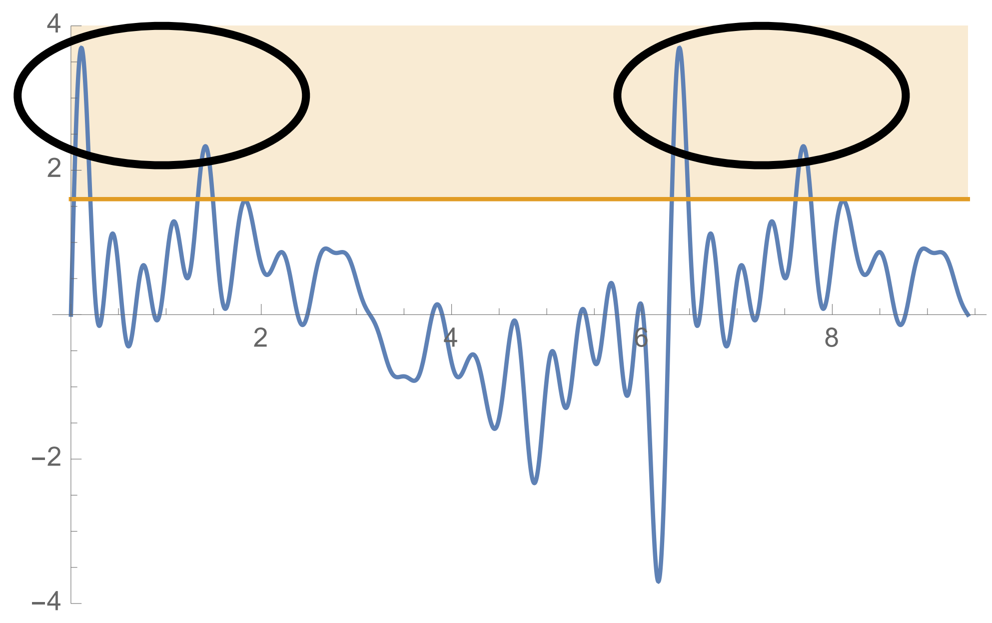
Halo Mass Function

- The number density of halos as a function of mass:



Halo Bias

- Halos are more likely to form on the peaks of the DM distribution:

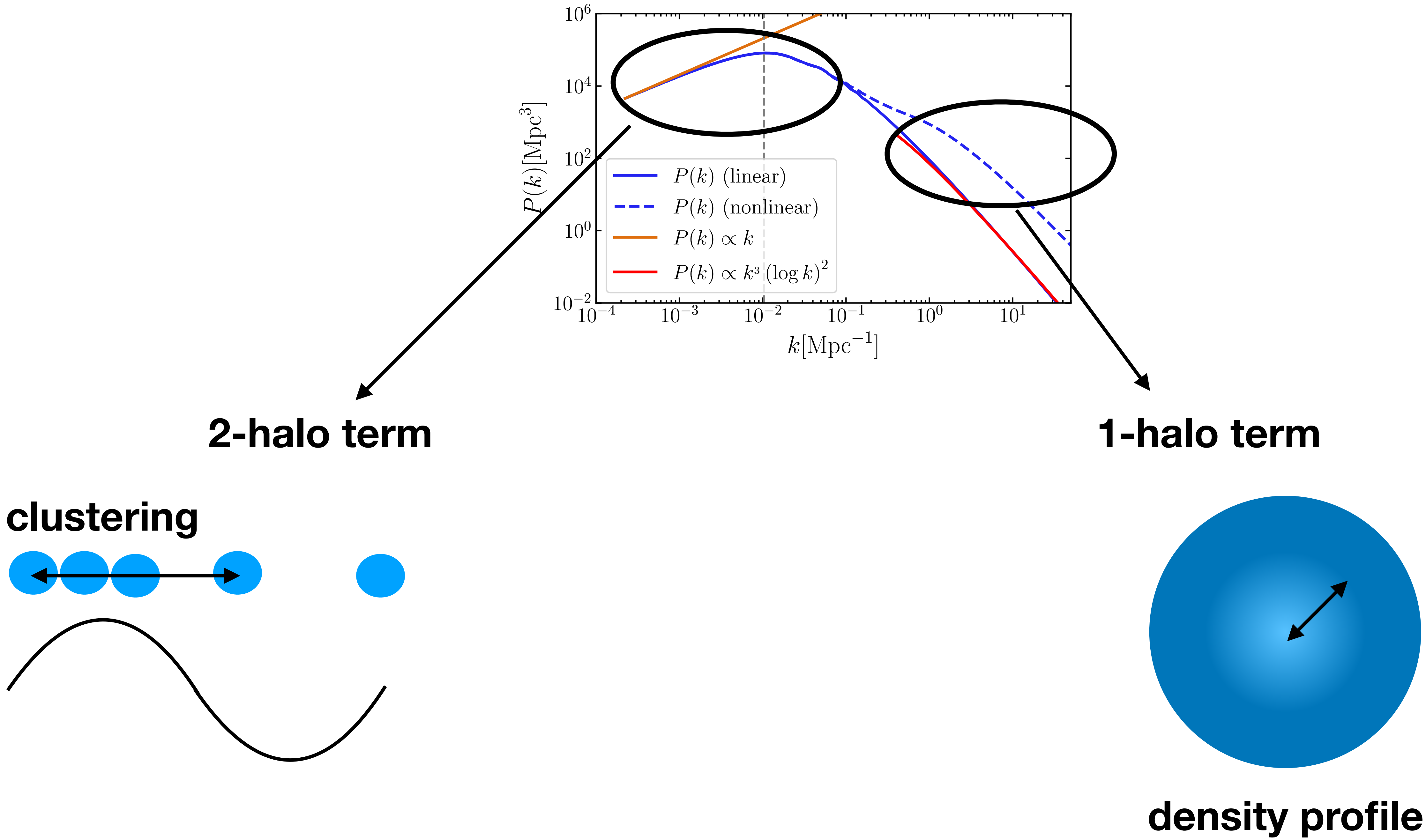


- Halos in a fixed mass range don’t form at random positions, but rather trace the underlying dark matter distribution.

$$P_h(k|m) = b_h(m)^2 P_m(k)$$

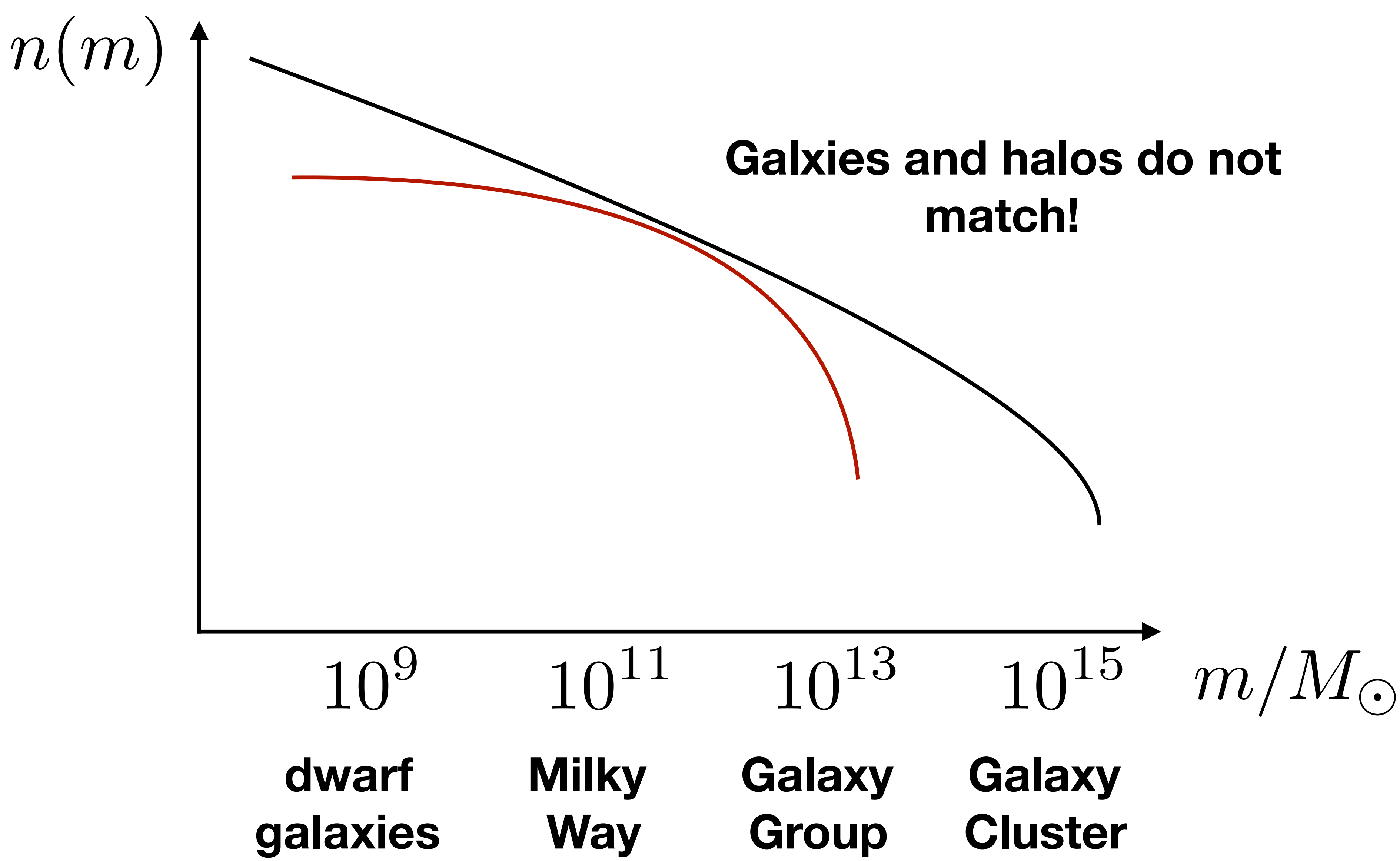
Halo Model of LSS

- If we assume all dark matter is bound inside halos:



Adding Galaxies

- Halo Occupation Distribution: prescription for adding galaxies to halos; assumed to depend primarily on halo mass.



Small-Scale issues with LCDM

- Core cusp problem: galaxy rotation curves of low-mass galaxies prefer a constant central dark matter density.
- Diversity problem: a larger than expected scatter in galaxy central densities inferred from rotation curves.
- Missing satellites problem: far fewer small galaxies in local group are observed than expected from the number of low-mass dark matter halos.
- Too-big-to-fail problem: Dwarf galaxies should have more stars than observed, based on their dark matter density.

Baryonic effects or new physics?

How does dark matter interact?

- Beyond gravity, does dark matter interact with itself or with baryonic matter?

DM is motivated by gravitational effects.

Why consider anything else?

- Compelling theory: dark matter is part of a more complete theoretical construction that necessarily predicts additional interactions.
- Empirical: observations require additional interactions.
- Nature is kind: without additional interactions, it will be difficult to determine what dark matter is.

Compelling theory?

- Best example: supersymmetry.
- Solves hierarchy problem, but includes weakly interacting massive particles (WIMPS) that can be the dark matter.

But SUSY in trouble….

- QCD Axion: can solve strong CP problem and serve as a dark matter candidate.
- Sterile neutrino: can be part of the mechanism responsible for neutrino masses.

In these scenarios, dark matter can interact with baryons and/or itself.

Empirical evidence?

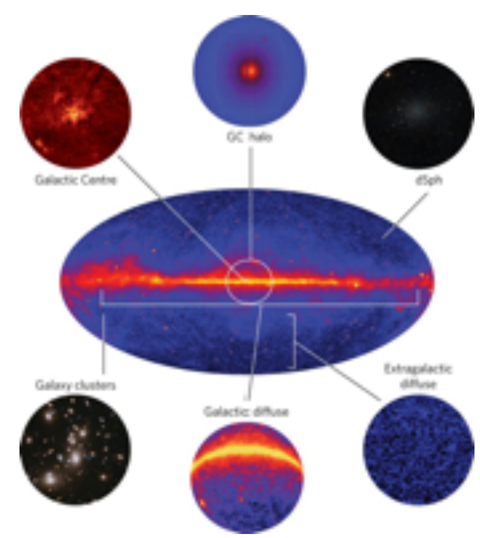
- Some of the small-scale problems with LCDM can be alleviated with self-interacting dark matter.

Could be baryonic physics…..

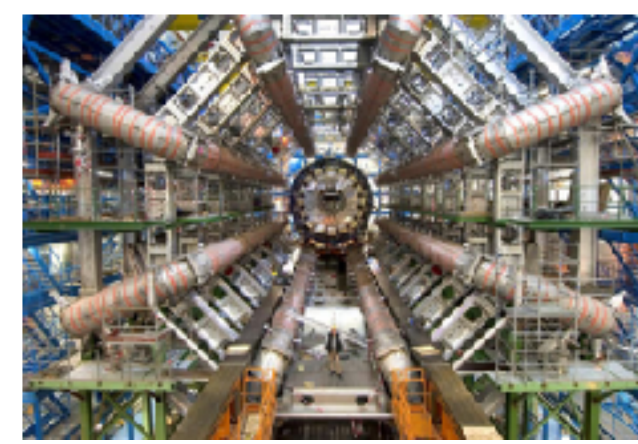
- Limits on interactions:

merging clusters

direct detection



indirect detection



direct production

And many more creative ways….

Nature is kind?

- I really hope that dark matter has non-gravitational interactions!
- If nature is kind: what interactions can we test given existing technology?

Can drive a creative cycle: we won’t know how to look if we don’t know what to look for.

Can be dangerous: if dark matter could be anything, and interact in any way, where do we stop?

What is dark matter?

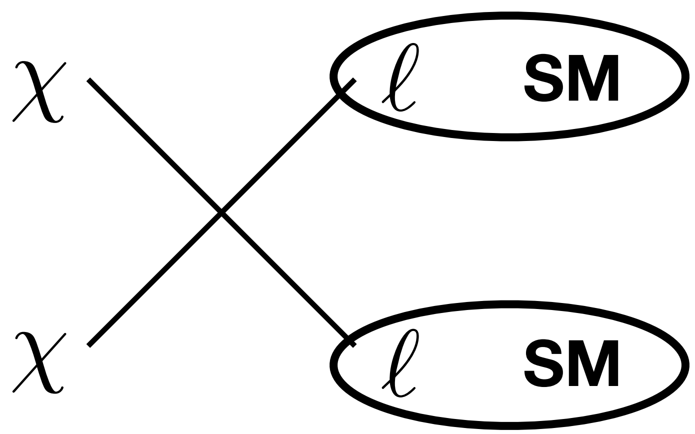
- Some possibilities:
    - Fundamental particle(s).
    - Composite particle(s).
    - Condensate(s).
    - Black holes.
    - ……….

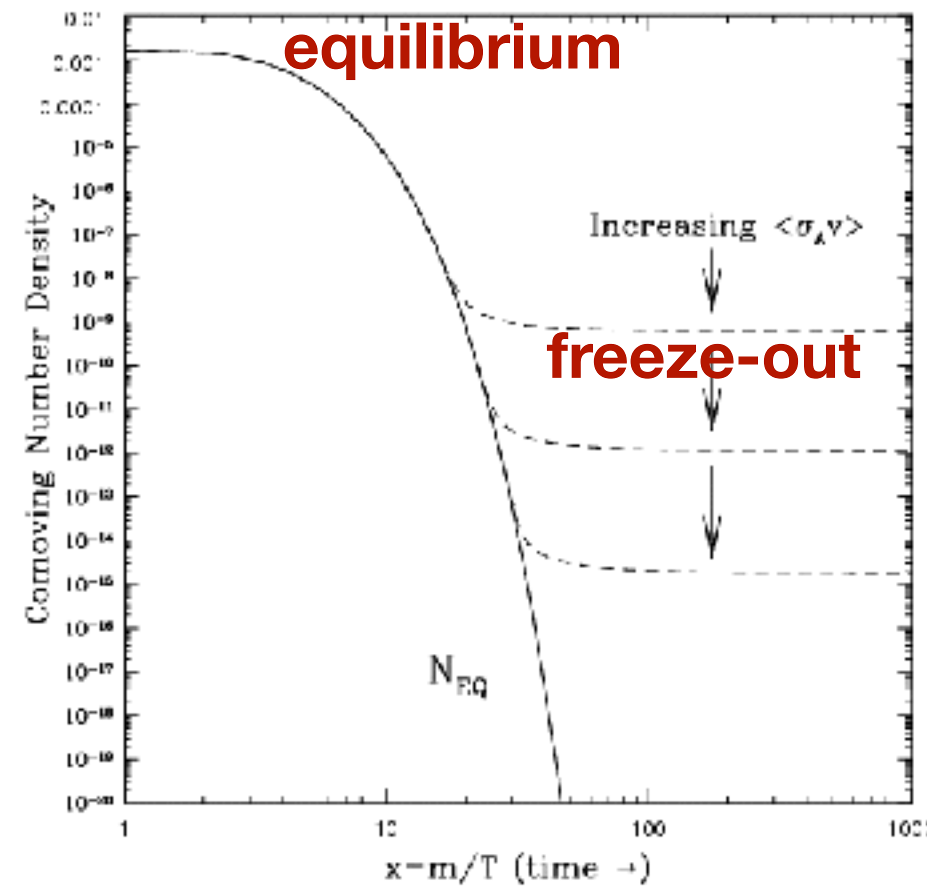
Important and interesting to look for observations that can distinguish these various candidates.

How was dark matter produced?

- A few possibilities:
  - Thermal production (e.g. WIMP)
  - Non-thermal production (e.g. Axion)

Thermal production





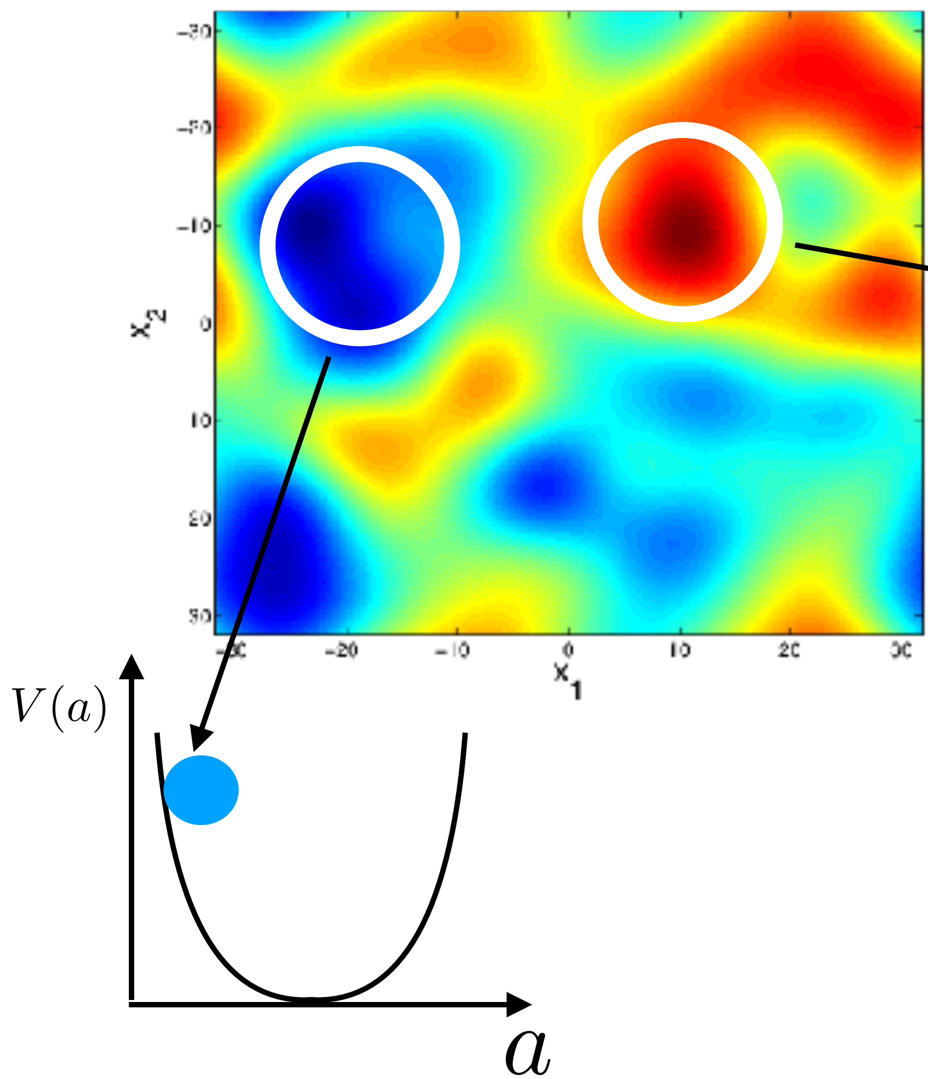
$$\frac{1}{a^3}\frac{d(a^3 n_X)}{dt} = \langle \sigma v\rangle \left[(n_X^{(0)})^2 - n_X^2 \right]$$

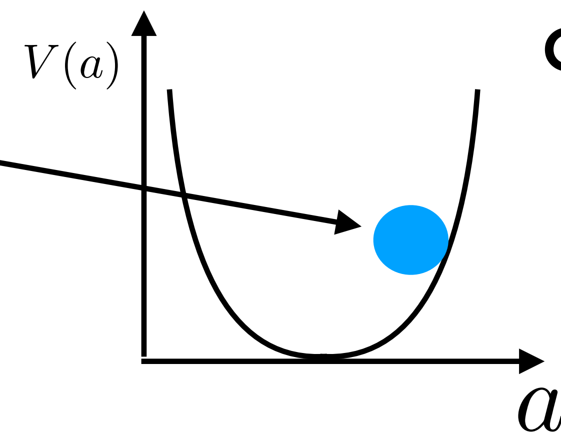
$$\Omega_\chi = 0.7 \left(\frac{m_\chi/T_f}{10}\right)\left(\frac{g_*(m_\chi)}{100}\right)^{1/2}\left(\frac{10^{-39}\ \mathrm{cm}^2}{\langle \sigma v\rangle}\right)$$

WIMP Miracle:

$$\Omega_\chi \sim 0.3 \qquad m_\chi \sim \mathrm{TeV} \qquad \langle \sigma v\rangle \sim 10^{-39}\ \mathrm{cm}^2$$

Non-thermal production



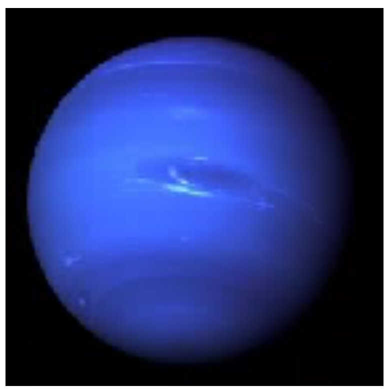


- Field is distributed randomly on ultra large scales

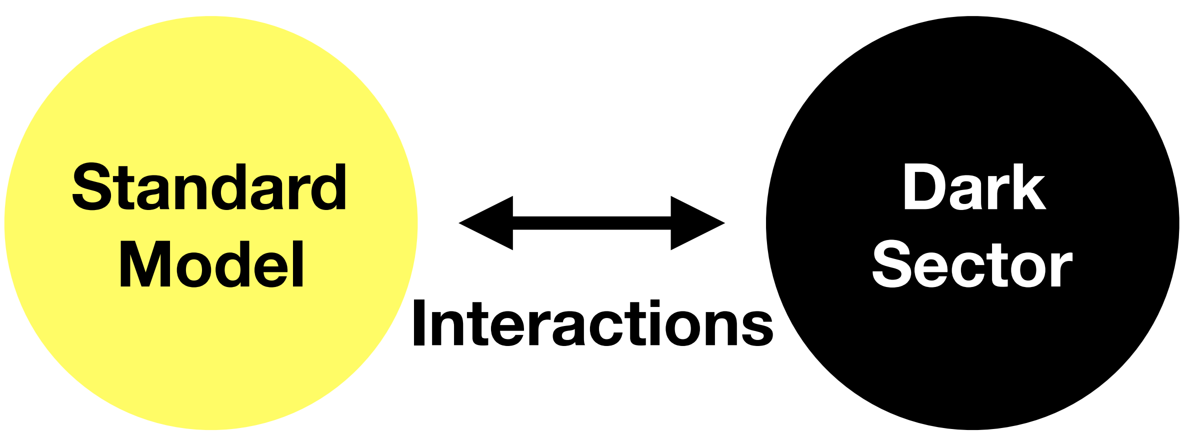
$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$

- At high temperatures, a is stuck.
- When $m > H$, a oscillates and dilutes like pressureless dust.
- Gravity and/or self interactions cause the condensate to fragment.

Some perspectives/opinions

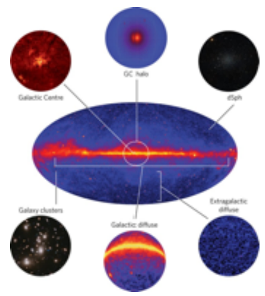


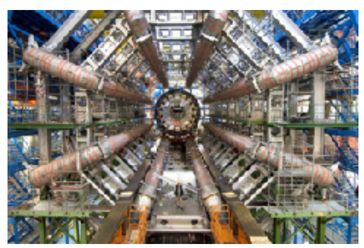
Should we modify gravity?

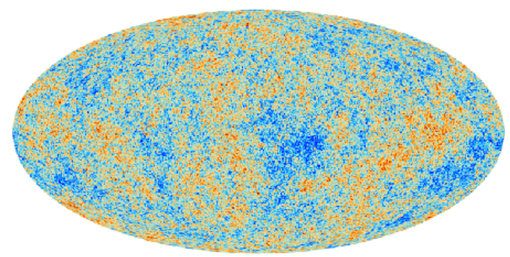


Is dark matter one component, or is there a dark sector?

Some perspectives/opinions







How will we identify what dark matter is?